PISTON ROD AND CONNECTING ROD

14.1. INTRODUCTION :

In I.C. engines, or steam engines, the force produced by the fuel gas or steam pushes the piston in the cylinder which causes the reciprocating movement of the piston and this reciprocating movement can be transmitted to the crank shaft of I.C. engines or wheel of steam engines through some intermediate elements such as piston rod and connecting rod as shown in figure 14.1. With respect to the places of applications and requirements these elements can be operated to convert the reciprocating motion of the piston into rotary motion of the crankshaft as in I.C. engines and steam engines or to convert the rotary motion into reciprocating motion as in the case of reciprocating compressors, pumps, power hammers and shaping machines etc.

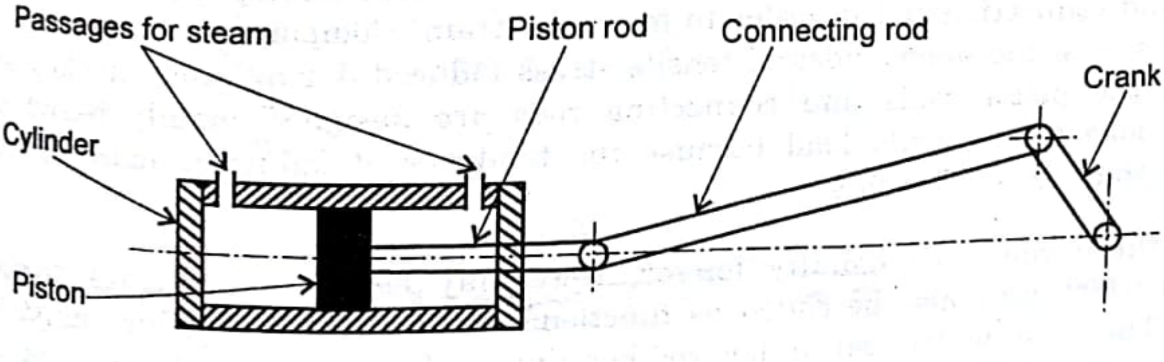


Fig. 14.1

14.2. MATERIALS REQUIRED :

When considering the constructional features of the above elements, the piston rod is usually having circular cross-section whereas the connecting rod is of I-shaped cross-section and these two rods are made longer in order to satisfy the work-requirement. Since these rods are subjected to axial loading, they should have sufficient strength to withstand the axial force applied on them without any failure. They are usually made of carbon steels or alloy steels of molybdenum and chromium with high tensile and compressive strengths.

14.3. STRUT OR COLUMN (An Analysis) :

Any machine part subjected to an axial compressive force is called a strut. A strut may be horizontal, inclined or vertical and the vertical strut is known as column. When a column or strut is subjected to a compressive load, it will fail at the ultimate compressive load called as crushing load. Sometimes for long column, it may bend or buckle at a lower load than crushing load and this buckling will affect the proper function of the column. This load for buckling is known as ***"buckling load or crippling load"*** which may be considered as design load for column or strut. Also the magnitude of buckling load for long column is relatively lower than that for short column.

The machine members that can be analysed based on column action are ***piston rods, valve push rods, connecting rod and screw jack.***

14.4. FORMULAS USED FOR THE DESIGN OF PISTON ROD AND CONNECTING ROD :

In I.C. engine the gas force produced during power stroke pushes the piston away from the cylinder head and this pushing force is transmitted to the connecting rod and due to this force, large amount of compressive stress is induced in the connecting rod. Similarly in the steam engine also, the steam force pushes the piston, piston rod and connecting rod in order to move the train. Comparing to this induced compressive stress, the magnitudes of tensile stress induced during other strokes are less. Hence the piston rods and connecting rods are designed mainly based on compressive load than tensile load because the tendency of failure is more during compression than tensile loading.

Since these rods are usually longer, they may bend (i.e., buckle) during compression which may also be called as functional failure and this failure must be prevented. The load which cause for rod-bending is known as buckling load or crippling load as mentioned earlier. This drawback of rod-bending will not occur during tensile loading. That is why the design of piston rod and connecting rod is carried out based on the compressive load especially buckling load (sometimes called as critical load).

For the design of piston rod and connecting rod, the following formulas can be employed.

(i) Euler's Formula.

(ii) Rankine's Formula.

(iii) Johnson's Formula.

The above three formulas are derived based on the characteristics of columns for their different end conditions, which are

1. Both the ends hinged.
2. Both the ends fixed.
3. One end is fixed and other end hinged.
4. One end is fixed and other end free.

The schematic diagrams for the above conditions are shown in figure 14.2.

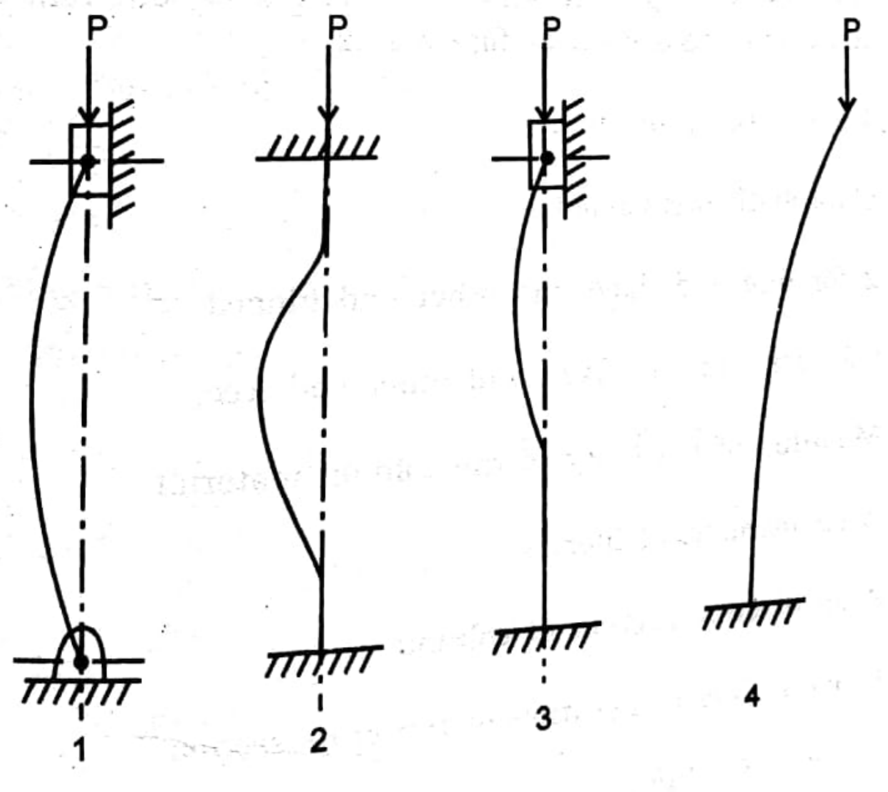


Fig. 14.2

Also the characteristics of columns depend on certain assumptions for proper functions. They are

1. Initially the column should be perfectly straight before loading and the load applied should be truely axial.
2. The cross-section of the column is uniform throughout its length.

3. They should have long length compared to their cross-section.

4. The functional failure occurs due to buckling alone.

Now based on the above assumptions, the Euler's formula, Rankine's formula and Johnson's formula are discussed as follows :

14.4.1. Euler's Formula :

Mr. Euler derived an expression for the crippling load (or buckling load) for long column under various end conditions as.

$$F_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 EAk^2}{l^2} = \frac{C\pi^2 EA}{\left(\frac{l}{k}\right)^2} \quad \text{.... (14.1)}$$

where, C = End fixity coefficient which is different for different end conditions as shown in figure 14.2

= 1 for both ends hinged

= 4 for both ends fixed

= 2 for one end fixed and other end hinged

= 0.25 for one end fixed and other end free.

E = Modulus of elasticity of the column material

I = Area moment of inertia

A = Area of cross-section of column

k = Least radius of gyration of the cross-section

l = Length of column

$\left(\frac{l}{k}\right)$ = Slenderness ratio (i.e., the ratio of length of column to the least radius of gyration)

Euler's formula can be expressed in another form also, which is given by,

$$F_{cr} = \frac{\pi^2 EI}{L^2} \quad \text{.... (14.2)}$$

14.4.2. Rankine's Formula :

This formula can be applied for both long columns and short columns. According to this formula, the crippling load, or buckling load is mathematically defined as

$$\frac{1}{F_{cr}} = \frac{1}{F_c} + \frac{1}{F_E} \quad \text{.... (14.3)}$$

where, F_{cr} = Crippling load defined by Rankine's formula

F_c = Ultimate crushing (i.e. compressive) load for the column.

$= S_c \cdot A$ (where S_c = Crushing stress (or) yield stress in compression and A = Area of cross-section of column).

$F_E \equiv$ Crippling load from Euler's formula $= \dfrac{\pi^2 EI}{L^2}$

By substituting $F_c = S_c \cdot A$ and $F_E = \dfrac{\pi^2 EI}{L^2}$ in the Rankine's formula (14.3), we get

$$F_{cr} = \frac{F_c F_E}{F_E + F_c} = \frac{F_c}{1 + \dfrac{F_c}{F_E}} = \frac{S_c \cdot A}{1 + \dfrac{S_c \cdot A \cdot L^2}{\pi^2 EI}} = \frac{S_c \cdot A}{1 + \dfrac{S_c \cdot A L^2}{\pi^2 EA k^2}}$$

$$= \frac{S_c \cdot A}{1 + \left(\dfrac{S_c}{\pi^2 E}\right)\left(\dfrac{L}{k}\right)^2} = \frac{S_c \cdot A}{1 + a\left\{\dfrac{L}{k}\right\}^2} = \frac{F_c}{1 + a\left(\dfrac{L}{k}\right)^2} \quad \text{.... (14.4)}$$

In the above formula, 'a' is considered as Rankine's constant $= \dfrac{S_c}{\pi^2 E}$ and this value may be different for different materials. The values of crushing stress and Rankine's constant are given in table 14.1.

Table 14.1

(Values of S_c & a)

S.No.	Material	S_c in N/mm^2	$a = \dfrac{S_c}{\pi^2 E}$
1.	Wrought iron	250	1/9000
2.	Mild steel	330	1/7500
3.	Cast iron	550	1/1600

14.4.3. Johnson's Formula :

Prof. Johnson derived a formula for the crippling load for short columns made of ductile materials keeping the column's both ends hinged. According to Johnson's formula, the crippling load is given by

$$F_{cr} = A\,S_y \left[1 - \frac{S_y}{4\,C\,\pi^2\,E} \left(\frac{L}{k} \right)^2 \right]$$

where,

S_y = Yield point stress $= 210\ N/mm^2$

C = End coefficient = 1 ($\because$ both ends hinged)

Usually the Johnson's formula is employed for the design of connecting rod.

14.5. DESIGN OF PISTON ROD :

The piston rod connected with the piston of an engine as shown in figure 14.3 transmits gas pressure developed by the fuel or steam to the crankshaft through connecting rod. Since one end of the piston rod is connected with piston by its tapered end with a nut and the other end is joined with the connecting rod through a cross-head by a cotter, its ends are having no revolving movement and so the piston rod may be considered as a column with fixed ends.

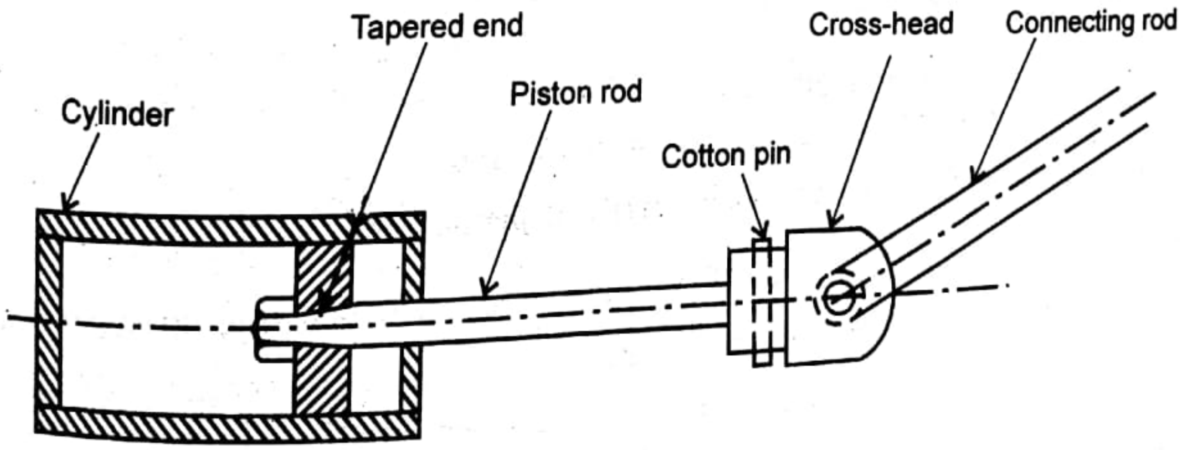


Fig. 14.3

Since the piston rod moves forwards and backwards in the engine cylinder due to gas force it will be subjected to alternate direct tensile and compressive forces and hence the piston rod is designed in such a way that it must sustain the load in tension and compression and also it should not buckle during compression. When the rod is considerably longer, it will buckle, eventhough it is having enough strength to overcome the failure by compression, which is undesirable. Hence the piston rod is designed mostly based on buckling load by using Euler's formula or Rankine's formula particularly when the slenderness ratio $\frac{l}{k}$ is greater than 40. When the ratio $\frac{l}{k} < 40$ then the size of rod is determined by simply comparing with the tensile or compressive strength to the applied load.

When designing the piston rod, the following steps may be observed.

1. First, the material of which the piston rod is to be made is selected and its design stresses in tension and compression are determined. Usually mild steel is adopted for making piston rod.

2. The force (F_1) produced by the fuel or steam, acting on the piston rod is determined using the equation,

$$F_1 = \frac{\pi}{4} \times D^2 \times p$$

where D = Diameter of piston

p = Pressure of steam or gas

3. This steam force is equated to the tensile or compressive strengths of piston rod (F_2), when the slenderness ratio, $\frac{l}{k} < 40$ as $F_1 = F_2$

where $F_2 = \frac{\pi}{4} d^2 (S_t)$ (or) $\frac{\pi}{4} d^2 (S_c)$

d = Diameter of piston rod

(S_t) = Allowable tensile stress of piston rod.

(S_c) = Allowable compressive stress of piston rod.

4. If $\frac{l}{k} \geq 40$, the applied force (F_1) is compared with buckling strength (F_{cr}) of piston rod using Euler's formula or Rankine's formula and especially the Euler's formula can be employed when $\frac{l}{k} > 80$.

$$\text{i.e. } F_{cr} = \frac{\pi^2 EI}{L^2} \text{ (or) } F_{cr} = \frac{S_c \cdot A}{1 + a\left(\frac{L}{k}\right)^2}$$

Usually, the buckling strength should be greater than the applied force for existence. Also, for the piston rod, since it is having fixed ends in both sides, the equivalent length, $L = \frac{l}{2}$ where l = Actual length of piston rod.

5. Using the above equations properly, the diameter of piston rod is determined.

14.6. DESIGN OF PUSH RODS :

In the I.C. engine the push rods are employed to open and close the inlet and outlet valves for the fuel entry or exit. Since the push rods use spherical seated bearings, they may be treated as pin end columns whose characteristics is similar to hinged end columns.

When the push rod is subjected to a load P, the design crippling load can be obtained as

$F_{cr} = P \times f_s$ where f_s is the factor of safety.

Then by using Euler's formula or Rankine's formula, the diameter of push rod may be evaluated. For the push rod made as hollow tube, the inner diameter of rod is usually taken as 0.8 times the outer diameter.

14.7. DESIGN OF CONNECTING ROD :

The connecting rod is employed as the intermediate element between the piston and crankshaft in I.C. engine or between the piston rod and crankshaft in the case of steam engine. It consists of an eye at the small end (so called due to its small size) for connecting the piston through piston pin, a long shank usually of I-section, and a big end, which is connected to the crankshaft pin, of split type and has a separate cap as shown in figure 14.4. The cap is secured to the body of the rod by means of two or four bolts. In large sized connecting rods, holes are provided for lubricating purposes.

The motion of the connecting rod is a combination of the rotation of the crank-end (i.e., big end) and the reciprocating motion of the piston end or small end. Therefore, the rod should be as light as possible to keep down the inertia forces. It must have sufficient strength to withstand the thrust of the piston.

The connecting rods for petrol engines are made of steel containing carbon 0.4 to 0.45% and manganese 1.4 to 1.8%. Material for diesel engine rods must have higher strength and hence steels containing chromium, nickel, molybdenum with 0.4 percent carbon are used. The connecting rods are usually made of drop-forging process. The properties of connecting rod materials are given in table 14.2.

Table 14.2: Connecting Rod Materials

Composition	S_u	S_y	S_e
Petrol engines			
1. Carbon steel			
(C = 0.37 – 0.45%, Mn = 0.5 – 0.8% + Si)	570 – 700	310 – 400	230 – 320
(C = 0.42 – 0.50%, Mn = 0.5 – 0.8% + Si)	600 – 750	340 – 430	250 – 300

Composition	S_u	S_y	S_e
2. Manganese steels (C = 0.41 – 0.49%, Mn = 1.4 – 1.8% + Si)	700 – 920	420 – 550	310 – 400
Diesel engines			
1. Chromium steel (C = 0.36 – 0.44%, Mn = 0.5 – 0.8%, Cr = 0.8 – 1.1% + Si)	750 – 1050	650 – 950	320 – 480
2. Alloy steel (C = 0.37 – 0.44%, Mn = 0.5 – 0.8%, Cr = 0.6 – 0.9%, Ni = 1.25 – 1.65%, Mo = 0.15 – 0.25% + Si)	1150 – 1700	850 – 1600	550 – 700

When the fuel force is transmitted from the piston rod to crank-shaft through connecting rod, the connecting rod is also subjected to alternate tensile and compressive forces. Here also the tendency of functional failure or breaking failure of the connecting rod is more during compression than during tension. Hence the compressive load, especially the buckling load may be taken as the design load for the connecting rod, similar to piston-rod design.

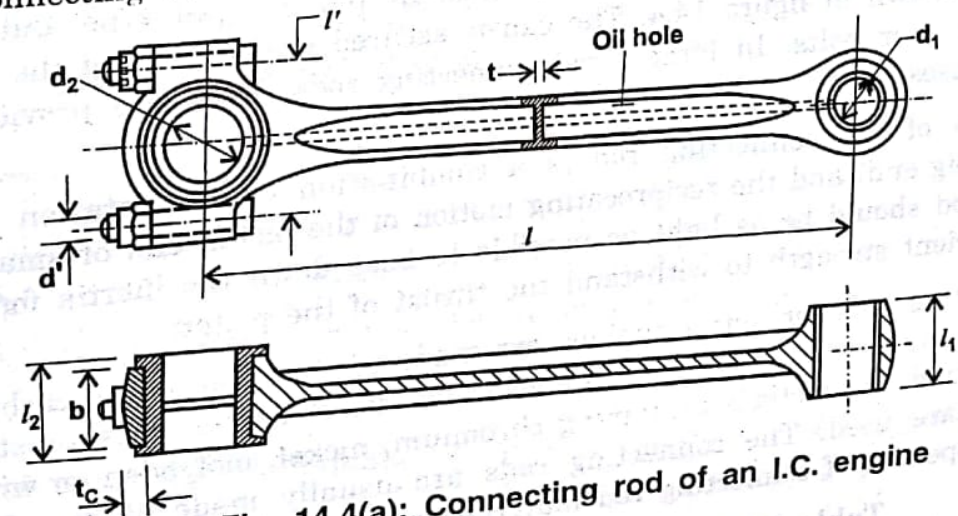


Fig. 14.4(a): Connecting rod of an I.C. engine

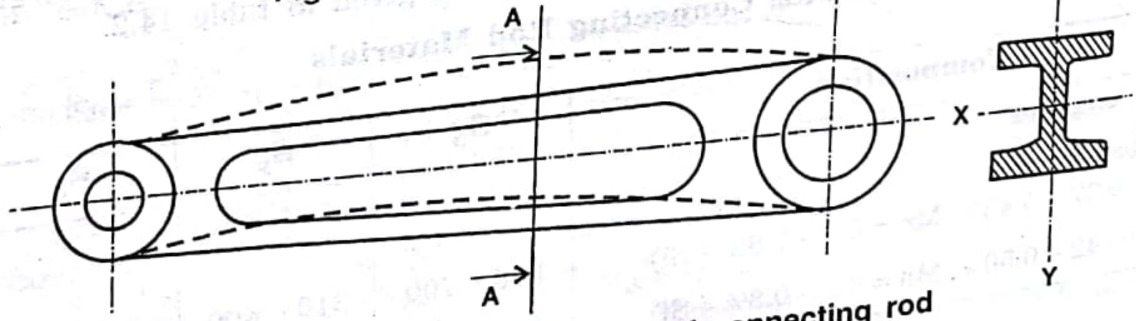


Fig. 14.4(b): Buckling of connecting rod

Generally, a connecting rod, subjected to an axial compressive load may buckle in two directions, i.e. in the plane of motion with x-axis as neutral axis or in the plane perpendicular to the plane of motion with y-axis as neutral axis. To overcome the buckling failure in these two directions, the connecting rod should be strong in both sections i.e. about x-axis and y-axis.

Determination of Shank Thickness :

For the design of connecting rod mostly Rankine's formula may be followed. Let us consider a I-section connecting rod with following parameters.

Let A = Cross-sectional area of the connecting rod.

l = Actual length of the connecting rod.

L = Equivalent length of the connecting rod.

S_c = Compressive yield stress (i.e., crushing stress)

F_{cr} = Crippling load (or) buckling load.

I_{xx} & I_{yy} = Area moment of inertia of the section about X-axis and Y axis respectively, and

k_{xx} & k_{yy} = Least radius of gyration of the section about X-axis and Y-axis respectively.

In general, the connecting rod is considered in such a way that its both ends are hinged for buckling about X-axis due to the relative rotation of the connecting rod with the piston pin and crank shaft and at the same time the both ends are assumed as fixed ends for buckling about Y-axis because of no relative movement. With these assumptions, the crippling load about X-axis (F_{crx}) and the crippling load about Y-axis (F_{cry}) can be computed according to Rankine's formula, as follows.

i.e., $$F_{crx} = \frac{S_c \cdot A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{S_c \cdot A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} \qquad (\because L = l \text{ for both ends hinged})$$

$$F_{cry} = \frac{S_c \cdot A}{1 + a\left(\frac{L}{k_{yy}}\right)^2} = \frac{S_c \cdot A}{1 + a\left(\frac{l}{2\,k_{yy}}\right)^2} \qquad \left(\because L = \frac{l}{2} \text{ for both ends fixed}\right)$$

In order to obtain the connecting rod equally strong in buckling about both the axes, its resisting strengths in both sections to the applied buckling load should be equal.

i.e. $$\frac{F_{crx}}{A} = \frac{F_{cry}}{A}$$

i.e., $$\frac{S_c}{1+a\left(\frac{l}{k_{xx}}\right)^2} = \frac{S_c}{1+a\left(\frac{l}{2\,k_{yy}}\right)^2}$$

Cancelling the common terms, we get

$$k_{xx}^2 = 4\,k_{yy}^2 \qquad \text{(A)}$$

$\because I_{xx} = A\,k_{xx}^2$ and $I_{yy} = A\,k_{yy}^2$, the equation (A) may be written as,

$$I_{xx} = 4\,I_{yy}.$$

From the above relation, we can understand that, if $I_{xx} = 4\,I_{yy}$, buckling will not occur about any axis. If $I_{xx} > 4\,I_{yy}$, buckling may occur about Y-axis. If $I_{xx} < 4\,I_{yy}$, buckling may occur about X-axis. But in actual practice I_{xx} is kept slightly less than $4I_{yy}$ so that the connecting rod should not buckle about Y-axis at any occasion eventhough it may be allowed to buckle very slightly about X-axis. The usual ratio of $\frac{I_{xx}}{I_{yy}}$ may be 3 to 3.5 and the proportions of various parts of mostly used I-section are given in **figure 14.5.** For this section, $\frac{I_{xx}}{I_{yy}} = 3.2$ which can be proved as follows :

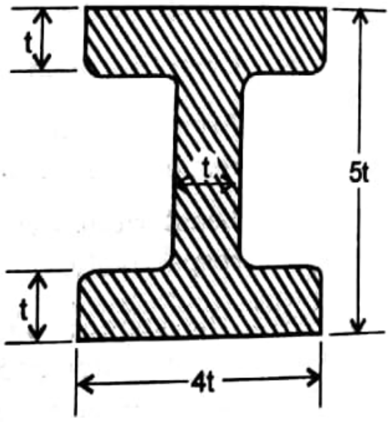


Fig. 14.5: Cross-section of connecting rod

I_{xx} = Moment of inertia about X-axis

$$= \frac{1}{12}\,[4\,t\,(5t)^3 - 3\,t\,(3t)^3] = \frac{419}{12}\,t^4$$

I_{yy} = Moment of inertia about Y-axis.

$$= \left[\frac{1}{12}\cdot(3t)\,t^3 + 2\times\frac{1}{12}\cdot t\,(4t)^3\right] = \frac{131}{12}\,t^4$$

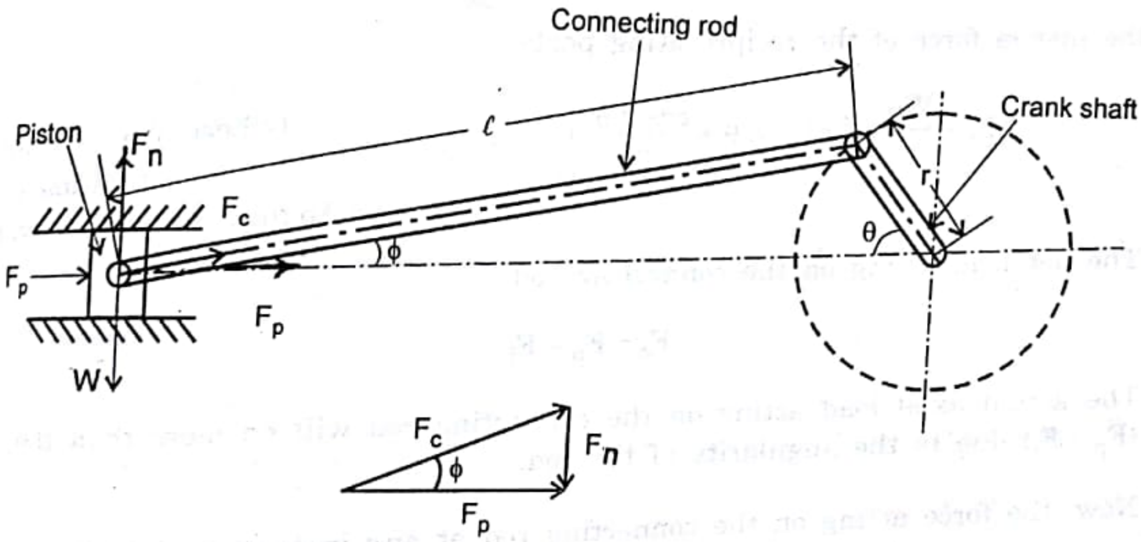
Now $$\frac{I_{xx}}{I_{yy}} = \frac{419}{12}\times t^4 \times \frac{12}{131\times t^4} = 3.2$$

Area of the section $= (4\,t\times 5\,t) - (3t\times 3t) = 11\,t^2$

The thickness, t, may be determined by equating the buckling strength of the connecting rod with the steam or gas force applied on it.

Force Analysis :

Since the force acting on the connecting rod is not truly axial as in piston rod, the connecting rod is subjected to ***inertia bending force*** apart from the ***direct steam or gas force***. For analysing the forces, let us consider the connecting rod as shown in figure 14.6.



Connecting rod of an I.C engine
Fig. 14.6

Let D = Diameter of the piston

A_p = Area of the piston

p = Pressure of gas (or) steam.

l = Length of connecting rod.

r = Radius of crankshaft

n = Ratio of length of connecting rod to the radius of crank $= \left(\frac{l}{r}\right)$

ϕ = Angle of inclination of the connecting rod with the line of stroke.

θ = Angle of inclination of crank from inner dead centre.

F_p = Force acting on the piston $= p \times A_p$

F_c = Force acting on the connecting rod.

F_i = Inertia force due to weight of the reciprocating parts.

The force acting on the piston due to pressure of gas or steam,

$$F_p = p \times A_p = p \times \frac{\pi}{4} D^2$$

and the inertia force of the reciprocating parts

$$F_i = \frac{W_R}{g} \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

(where W_R = Weight of reciprocating parts
ω = Angular velocity of crank)

The net load acting on the connecting rod

$$F_c = F_p - F_i$$

The actual axial load acting on the connecting rod will be more than the net load $(F_p - F_i)$ due to the angularity of the rod.

Now, the force acting on the connecting rod at any instant is given by

$$F_c = \frac{F_p - F_i}{\cos\phi}$$

Normally, the inertia force due to the weight of reciprocating parts is very small (i.e., about 5% of gas force), it can be neglected when designing the connecting rod.

Hence $F_c = \dfrac{F_p}{\cos\phi}$

Since the piston is under reciprocating action, the connecting rod will be subjected to maximum force when the crank angle $\theta = 90°$ and for other positions, the force values are reduced and for $\theta = 0°$ and $\theta = 180°$, the forces are zeros. Also, the inclination of the connecting rod, $\phi = \phi$ (max) when $\theta = 90°$. Hence the maximum force acting on the connecting rod, is given by

$$F_{c\,(max)} = \frac{F_p}{\cos\phi_{(max)}} = \frac{F_p}{\left(\frac{\sqrt{l^2 - r^2}}{l}\right)} = \frac{F_p}{\sqrt{1 - \left(\frac{1}{n}\right)^2}}$$

In general, n should be at least 3

Hence, for $n = \frac{l}{r} = 3$, $F_c = \frac{F_p}{\sqrt{1-\left(\frac{1}{3}\right)^2}} = 1.06\ F_p$

for $n = 4$, $F_c = \frac{F_p}{\sqrt{1-\left(\frac{1}{4}\right)^2}} = 1.03\ F_p$

for $n = 5$, $F_c = \frac{F_p}{\sqrt{1-\left(\frac{1}{5}\right)^2}} = 1.02\ F_p$

Since the maximum deviation of force from the steam force (F_p) is only 6%, the force acting on the connecting rod may be taken equal to the steam force (F_p) by neglecting that small deviation of force for design purpose.

i.e. $F_c = F_p$

Also the maximum bending moment due to inertia bending force is given by the relation.

$$M_{max} = m \times \omega^2 r \times \frac{l}{9\sqrt{3}} \qquad \text{.... (14.5)}$$

where m = Mass of connecting rod

ω = Angular speed in rad /s

l = Length of connecting rod

r = Radius of the crank

The maximum bending stress $= \frac{M_{max}}{Z}$

where Z = Section Modulus.

Dimensions of Connecting rod Ends :

Now the other parts of connecting rod such as its small end, big end and bolts are designed as follows.

The small end is made as solid eye without any split and is provided with brass bushes inside the eye and the big end is split and the top cap is joined with the remaining parts of connecting rod by means of bolts as shown in figure 14.4. By this set up the connecting rod can be dismantled without removing the crank shaft. In the big end also, the brass bushes of split type are employed.

The parameters of small end and big end are determined based on the bearing pressures.

Let

l_1, d_1 = Length and diameter of piston pin (i.e., small end) respectively.

l_2, d_2 = Length and diameter of crank pin (i.e., big end) respectively.

p_{b1}, p_{b2} = Design bearing pressures for the small end and big end respectively.

Bearing load applied on the piston pin (i.e., small end) is given by,

$$F_1 = p_{b1} \cdot l_1 d_1$$

and the bearing load applied on the crank pin (i.e., big end) is given by,

$$F_2 = p_{b2} \cdot l_2 d_2$$

Usually, the design bearing pressures for the small end and big end may be taken as,

$p_{b1} = 12.5$ to $15.4\ N/mm^2$ and $p_{b2} = 10.8$ to $12.6\ N/mm^2$

Similarly, the ratio of length to diameter for small end and big end may be assumed as,

$\frac{l_1}{d_1} = 1.5$ to 2 and $\frac{l_2}{d_2} = 1.0$ to 1.25

Usually, low design stress value is selected for big end than that for small end.

The highest load to be carried by these bearings containing piston pin and crank pin is the maximum compressive load produced by the gas pressure neglecting the inertia force due to its small value.

At the same time, the bolts are designed based on the inertia force because the maximum load applied on the bolts is the inertia force of the reciprocating parts which is given by

$$\text{Inertia force, } F_i = m\,\omega^2\,r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$n = \frac{l}{r} = \frac{\text{Length of connecting rod}}{\text{Crank radius}}$$

The maximum inertia force will be obtained when the crank shaft is at dead centre position,

i.e., at $\theta = 0$

$$\therefore \; F_{im} = m\,\omega^2\,r\left(1+\frac{1}{n}\right)$$

By equating this maximum inertia force to the tensile strength of bolts at their core diameters, the size of bolts may be determined.

i.e., For two bolts,

$$F_{im} = 2\times\frac{\pi}{4}\,d_c^2\times S_t$$

The nominal diameter may be selected from the manufacturer's table. (Usually $d_c = 0.84\,d$ where d is the nominal diameter)

The cap is usually treated as a beam freely supported at the bolt centres and loaded in a manner intermediate between uniformly distributed load and centrally concentrated loaded.

Maximum bending moment at the centre of cap is given by,

$$M = \frac{Wl'}{6}$$

where W = Maximum load equal to inertia force of reciprocating parts $= F_{im}$

Hence $M = \dfrac{F_{im}\,l'}{6}$

l' = Distance between bolt centres

= Diameter of crank pin + (2 × wall thickness of bush) + diameter of bolt + some extra marginal thickness

Width of cap may be calculated as,

b = Length of crank pin − 2 × flange thickness of bush

Usually, the wall thickness and flange thickness of bush may be taken as about 5 mm.

Bending stress induced in the cap,

$$S_{bc} = \frac{M}{Z}$$

where Z = Section modulus of the cap

$= \frac{1}{6}\cdot b\,t_c^2$

t_c = Thickness of cap.

By comparing this induced bending stress with the design stress, the thickness of cap may be evaluated.

14.8. DESIGN PROCEDURE FOR CONNECTING ROD :

For the design of connecting rod, the following steps may be observed.

1. From the statement of problem, note the pressure of steam or gas, length of connecting rod, crank radius etc. Then, select suitable material usually mild steel for the connecting rod and find its design stresses. Assume the essential non-given data suitably based on the working conditions.

2. Select I-section connecting rod if possible and determine its moment of inertia about X-axis and Y-axis.

3. Equate the steam force with buckling strength of connecting rod using Rankine's formula and determine the dimensions of connecting rod.

4. Calculate the maximum bending stress and then compare it with design stress of the connecting rod for checking.

14.9. SOLVED PROBLEMS :

Problem 14.1 :

Design a piston – rod for an engine whose bore diameter is 200 mm. The length of piston rod is 750 mm and the stroke length is 400 mm. The pressure of steam is found to be 1 N/mm². Assume factor of safety as 5.

Solution : Bore diameter of the engine (ie Diameter of the piston)

$$D = 200 \text{ mm.}$$

Length of piston rod, $l = 750$ mm.

Stroke length, $= 400$ mm

$\therefore$ Crank radius, $r = \dfrac{\text{Stroke length}}{2} = 200$ mm

Pressure of steam $p = 1 \text{ N/mm}^2$

Factor of safety, $f_s = 5$

Steam force acting on the piston, $F_1 = \dfrac{\pi}{4} D^2 \times p$

$$= \frac{\pi}{4} \times 200^2 \times 1 = 31416 \text{ N}$$

Now the design buckling load $F_{cr} = F_1 \times f_s$

$$= 31416 \times 5 = 157080 \text{ N}$$

Let d = Diameter of piston rod.

Area moment of inertia of piston rod $I = A\,k^2$

where A = Area of cross-section of piston rod

and k = Its least radius of gyration.

Let the piston rod may be circular in cross section

$$\text{Now } k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

Since, k is the function of d, we can not find the slenderness ratio $\left(\frac{l}{k}\right)$ without knowing the value of d, in order to know whether the rod may be a short column type or long column type. Hence it is better to use Rankine's formula which will suit for both types of columns. According to Rankine's formula the buckling load is given by,

$$F_{cr} = \frac{S_c \cdot A}{1 + a\left(\frac{L}{k}\right)^2}$$

Let the piston rod is made of mild steel and hence its crushing stress, $S_c = 330 \text{ N/mm}^2$ and $a = \frac{1}{7500}$.

Since the both ends of the piston rod are assumed as fixed ends, the equivalent length of rod, $L = \frac{l}{2} = \frac{750}{2} = 375$ mm.

Substituting the above values in the Rankine's formula we get,

$$157080 = \frac{330 \times \frac{\pi d^2}{4}}{1 + \frac{1}{7500}\left(\frac{375 \times 4}{d}\right)^2} = \frac{260 d^2}{1 + \frac{300}{d^2}} = \frac{260 d^4}{d^2 + 300}$$

(or) $260 d^4 - 157080 d^2 - 47124 \times 10^3 = 0$

i.e, $d^4 - 604 d^2 - 181246 = 0$

$$\therefore d^2 = \frac{604 + \sqrt{604^2 + (4 \times 181246)}}{2} = 824 \text{ mm}^2$$

$\therefore d = 28.7$ mm

Take $d = 30$ mm

Diameter of piston rod = 30 mm. **(Answer)**

Problem 14.2 :

The maximum load on a hollow push rod of a petrol engine is 1500 N. The length of rod is 300 mm and the ratio of outer diameter to inner diameter is 1.25. Spherical seated bearings are used for the push rod. For the factor of safety of 3, design the push rod. Assume the modulus of elasticity of push rod material is 210 kN/mm^2.

Solution :

Given :

Load acting on the push rod, P = 1500 N

Length of push rod, $l = 300$ mm.

Ratio of diameters, $\frac{d_0}{d_i} = 1.25$

Factor of safety, $f_s = 3$.

Modulus of elasticity $E = 210 \text{ kN/mm}^2 = 210 \times 10^3 \text{ N/mm}^2$

Now,

The crippling load for the push rod, $F_{cr} = P \times f_s = 1500 \times 3 = 4500$ N

According to Euler's formula, the crippling load is given by

$$F_{cr} = \frac{\pi^2 E I}{L^2}$$

where I = Area moment of inertia of push rod

$$= \frac{\pi}{64}\left(d_0^4 - d_i^4\right) = \frac{\pi}{64} d_0^4 \left[1 - \left(\frac{d_i}{d_0}\right)^4\right]$$

$$= \frac{\pi}{64} d_0^4 \left[1 - \left(\frac{1}{1.25}\right)^4\right] = 0.03\, d_0^4$$

L = Equivalent length of push rod = l = 300 mm

($\because L = l$ for pin ends)

Substituting these values in Euler's formula, we get

$$4500 = \frac{\pi^2 \times 210 \times 10^3 \times 0.03\, d_0^4}{300^2}$$

(or) $d_0 = \left(\dfrac{4500 \times 300^2}{\pi^2 \times 210 \times 10^3 \times 0.03} \right)^{1/4} = 8.98 \text{ mm} = 9 \text{ mm}$

$\therefore d_i = \dfrac{d_0}{1.25} = \dfrac{9}{1.25} = 7.2 \text{ mm}$

Inner diameter of push rod $= 7.2$ mm.

Outer diameter of push rod $= 9.0$ mm.

Length of push rod $= 300$ mm **(Answer)**

Problem 14.3 :

The connecting rod of a petrol engine is to be designed for the following data.

Piston diameter	**– 80 mm.**
Stroke	**– 120 mm.**
Weight of reciprocating parts	**– 15 N**
Length of connecting rod	**– 240 mm.**
Speed (Maximum)	**– 2800 rpm.**

Explosion pressure corresponding to 10° of crank angle is 3 MPa.

Factor of safety – 6

If the connecting rod is to be made of 40 Cr1 steel, find the dimensions of the I-section connecting rod. **(Madras University)**

Solutions : (Refer figure 14.4(a))

Given :

Piston Diameter, $D = 80$ mm

Stroke, $2r = 120$ mm

$\therefore$ Crank radius, $r = 60$ mm

Length of connecting rod, $l = 240$ mm.

Pressure of the gas, $p = 3 \text{ MPa} = 3 \times 10^6 \text{ N/m}^2 = 3 \text{ N/mm}^2$

Factor of safety, $f_s = 6$

Maximum speed $N = 2800$ rpm.

Force acting on the piston due to gas pressure.

$$F_p = \frac{\pi}{4} D^2 \times p = \frac{\pi}{4} \times 80^2 \times 3 = 15080 \text{ N}$$

Design load acting on the piston, $F_d = F_p \times f_s$

$$= 15080 \times 6 = 90480 \text{ N}$$

(i) Design of connecting rod shank :

Let t = Web thickness of connecting rod shank

Since the connecting rod is made of 40 Cr1 steel, its crushing stress may be assumed as, $S_c = 330 \text{ N/mm}^2$. Also, consider the given I-section connecting rod is having proportions as shown in figure 14.5 for various parts in order to allow buckling about X-axis and not to allow buckling about Y-axis.

Now area of cross section

$$A = 2\,(4t \times t) + 1\,(t \times 3t) = 11\,t^2$$

Moment of inertia about x-axis,

$$I_{xx} = \frac{1}{12}\,(4t)\,(5t)^3 - \frac{1}{12}\,3\,t\,(3t)^3 = \frac{419\,t^4}{12} = 35\,t^4 \text{ mm}^4$$

Least radius of gyration about X-axis $k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{35\,t^4}{11\,t^2}} = 1.78\,t$

We have already known that the inertia force due to receiprocating mass is having very little effect on the connecting rod, we can assume that the rod can be designed against gas force only. That is, the design buckling load is taken as,

$$F_{cr} = 90480 \text{ N}$$

Now, according to Rankine's formula, the buckling load (or) crippling load is obtained as

$$F_{cr} = \frac{S_c \cdot A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

For buckling about X-axis, the both ends of connecting rod are assumed as hinged ends and hence the equivalent length of rod, $L = l = 240$ mm.

Hence, $90480 = \dfrac{330 \times 11\, t^2}{1 + \dfrac{1}{7500}\left(\dfrac{240}{1.78\, t}\right)^2}$ $\left(\because a = \dfrac{1}{7500} \text{ for mild steel i.e. for 40 Cr1 steel} \right)$

$$= \frac{3630\, t^2}{1 + \dfrac{2.42}{t^2}} = \frac{3630\, t^4}{t^2 + 2.42}$$

(or) $3630\, t^4 - 90480\, t^2 - 218962 = 0$

(or) $t^4 - 25\, t^2 - 60 = 0$

$\therefore t^2 = \dfrac{25 + \sqrt{25^2 + 4\,(60)\,(1)}}{2} = 27.2$

$\therefore t = 5.22$ mm.

Let the thickness of web, $t = 6$ mm.

Height of connecting rod $= 5t = 30$ mm.

Width of connecting rod $= 4\,t = 24$ mm

Length of connecting rod $= 240$ mm (given)

Now the maximum bending moment acting on the rod,

$$M_{max} = m\, \omega^2\, r \times \frac{l}{9\sqrt{3}}$$

where m = Mass of the connecting rod in kg

= Volume $\times$ density = Area $\times$ length $\times$ density

$= 11\, t^2 \times l \times \rho = 11 \times 6^2 \times 240 \times \dfrac{7800}{10^9} = 0.75$ kg

$\left(\text{Assuming } \rho = 7800 \text{ kg/m}^3 = \dfrac{7800}{10^9} \text{ kg/mm}^3 \right)$

ω = Angular speed of the crank

$= \dfrac{2\,\pi\, N}{60} = \dfrac{2\,\pi \times 2800}{60} = 293$ rad/s

r = Radius of the crank $= 60$ mm

$$\therefore M_{max} = 0.75 \times 293^2 \times 60 \times \frac{240}{9\sqrt{3}} \times \frac{1}{10^3}$$

$$= 59.5 \times 10^3 \text{ N–mm}$$

Section modulus, $Z_{xx} = \frac{I_{xx}}{\frac{5t}{2}} = \frac{419 t^4}{12} \times \frac{2}{5t} = 14 t^3 = 14 \times 6^3 = 3 \times 10^3 \text{ mm}^3$

Maximum bending stress,

$$S_{b\,max} = \frac{M_{max}}{Z_{xx}} = \frac{59.5 \times 10^3}{3 \times 10^3} = 19.8 \text{ N/mm}^2$$

Since this is less than compressive crushing stress 330 N/mm^2, our design is safe.

(ii) Design of connecting rod ends :

The small end of the connecting rod is to be connected with the piston pin and the big end with the crank pin and hence the dimensions of connecting rod ends are decided by the piston pin and crank pin.

Let l_1, d_1 = Length and diameter of piston pin.

l_2, d_2 = Length and diameter of crank pin.

p_{b1}, p_{b2} = Design bearing pressure for piston pin and crank pin

F_1, F_2 = Bearing load applied on piston pin and crank pin = Gas load (F_p)

Now

$$F_1 = p_{b1} \times l_1 \times d_1 \text{ and } F_2 = p_{b2} \times l_2 \times d_2$$

Assuming $p_{b1} = 14$ N/mm^2 and $l_1 = 1.5\, d_1$

we get,

$$F_1 = p_{b1} \times 1.5 d_1 \times d_1 = F_p$$

$$\text{(or)}\ d_1 = \left(\frac{F_p}{p_{b1} \times 1.5}\right)^{1/2} = \left(\frac{15080}{14 \times 1.5}\right)^{1/2} = 26.8 \text{ mm} = 30 \text{ mm (say)}$$

$$\therefore l_1 = 1.5\, d_1 = 1.5 \times 30 = 45 \text{ mm.}$$

Similarly, by assuming $p_{b2} = 11\ N/mm^2$ and $l_2 = d_2$,

we get, $F_2 = p_{b2} \times d_2 \times d_2 = F_p$

$$(or)\ d_2 = \left(\frac{F_p}{p_{b2} \times 1.0}\right)^{1/2} = \left(\frac{15080}{11 \times 1.0}\right)^{1/2} = 37\ mm = 40\ mm\quad (say)$$

$$\therefore l_2 = d_2 = 40\ mm$$

Now, the inner diameter of small end, $D_{is} = d_1 = 30$ mm

Outer diameter of small end $D_{os} = D_{is} + (2 \times \text{bush thickness}) + (2 \times \text{marginal thickness})$

$$= 30 + (2 \times 5) + (2 \times 5) = 50\ mm.$$

Similarly for big end,

Inner diameter, $D_{ib} = d_2 = 40$ mm.

Outer diameter, i.e., Length of big end $D_{ob} = D_{ib} + 2 \times \text{bush thickness} + (2 \times \text{bolt diameter}) + (2 \times \text{marginal thickness})$

Since bolt diameter is not known, D_{ob} can not found out at present.

(iii) Design of bolts :

Bolts are subjected to inertia force by the reciprocating parts.

Inertia force, $F_i = m\,\omega^2\,r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

Maximum inertia force is obtained when $\theta = 0$ which is given by,

$$F_{im} = m\,\omega^2\,r\left(1 + \frac{1}{n}\right)$$

$$m = \frac{W_r}{g} = \frac{15}{9.81} = 1.53\ kg$$

$$\omega = \frac{2\pi N}{60} = \left(\frac{2\pi \times 2800}{60}\right) = 293.2\ rad/s$$

$$r = 60\ mm$$

$$n = l/r = \frac{240}{60} = 4$$

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Substituting we get,

$$F_{im} = 1.53 \times 293.2^2 \times \frac{60}{10^3}\left(1 + \frac{1}{4}\right) = 9865 \text{ N}$$

Let d_c = Core diameter of bolt

S_t = Allowable tensile stress for bolt material

n' = Number of bolts

Now $S_t = \frac{\text{Yield stress in tension}}{\text{Factor of safety}} = \frac{S_y}{f_s} = \frac{600}{6} = 100 \text{ N/mm}^2$

($\because S_y = 600 \text{ N/mm}^2$ Assumed)

$n' = 2$

Now equating $F_{i(max)}$ to bolt strength, we get

$$F_{im} = 2 \times \frac{\pi}{4} d_c^2 \times S_t$$

(or) $d_c = \left(\frac{4 \times F_{im}}{2\pi S_t}\right)^{1/2} = \left(\frac{4 \times 9865}{2\pi \times 100}\right)^{1/2} = 7.92 \text{ mm}$

$\therefore$ Nominal diameter of bolt, $d_b = \frac{d_c}{0.84} = 9.4$ mm. = 10 mm (Standard)

Hence M10 bolt may be selected.

Now the outer diameter of big end bearing,

$D_{ob} = D_{ib}$ + (2 × bush thickness) + (2 × bolt thickness) + (2 × marginal thickness)

$= 40 + (2 \times 5) + (2 \times 10) + (2 \times 10) = 90$ mm.

(iv) Design of Cap :

Bending stress induced in the cap is given by

$$S_{bc} = \frac{M}{Z} = \frac{W l'}{6} \times \frac{6}{b t_c^2} = \frac{W l'}{b t_c^2}$$

Now W = Maximum load applied on the cap

= Inertia force ($F_{i\,(max)}$) = 9865 N

l' = Distance between bolt centres

= Diameter of crankpin + (2 × bush thickness) + diameter of bolt + (2 × marginal thickness)

= 40 + (2 × 5) + 10 + (2 × 7.5) = 75 mm.

b = Width of cap

= Length of crankpin – (2 × Flange thickness of bush)

= 40 – (2 × 5) = 30 mm.

$$\therefore S_{bc} = \frac{W\,l'}{b t_c^2}$$

$$\text{(or)}\ t_c = \left(\frac{W\,l'}{bS_{bc}}\right)^{1/2} = \left[\frac{9865 \times 75}{30 \times 100}\right]^{1/2} \qquad (\text{Assume } S_{bc} = 100\ \text{N/mm}^2)$$

$$= 15.7\ \text{mm} = 16\ \text{mm (say)}$$

Specifications :

1. Thickness of shank web = 6 mm.
2. Height of shank = 30 mm.
3. Width of shank = 24 mm.
4. Inner diameter of small end = 30 mm.
5. Outer diameter of small end = 50 mm.
6. Inner diameter of big end = 40 mm.
7. Outer diameter of big end = 90 mm.
8. Length of connecting rod = 240 mm.
9. Diameter of bolt = 10 mm
10. Number of bolts = 2
11. Width of cap = 30 mm
12. Thickness of cap = 16 mm.

CRANKSHAFT

15.1 INTRODUCTION :

Crankshaft is another essential component of the engine which receives steam or fuel force from connecting rod and converts the reciprocating motion of the connecting rod into a rotary motion. It is the most complicated and strained engine part which is subjected to cyclic loads due to gas pressure, inertia forces and their couples. In addition, there may exist bending moment caused by the weight of the flywheel coupled to the engine. The crankshaft consists of three main parts namely (i) ***Shaft part or journal*** which revolves in the main bearing, (ii) ***Crank-pin*** with which the big end of the connecting rod is linked and (iii) ***Crank-web*** (also known as cheek) which unites the crank pin with the journal. Smaller engines use one piece forged crankshafts with cast-iron counter-weights bolted to the crank. In large engines, the shaft, crank and pin may be made separately and then assembled.

When the force is transmitted from the connecting rod to the crank-shaft, all parts of crank-shaft will not be subjected to same kind and same magnitudes of stresses. They may vary from position to position with respect to line of stroke. The stresses which arise in a crankshaft are from bending and twisting moments.

15.2 TYPES OF CRANK-SHAFT :

Crankshaft may be classified into two types namely

(a) Over hung crankshaft or side crank-shaft.

(b) Supported crankshaft or centre crankshaft.

The centre crank-shaft is further divided as

(i) Single throw or single crank type.

(ii) Multi throw or multi crank type.

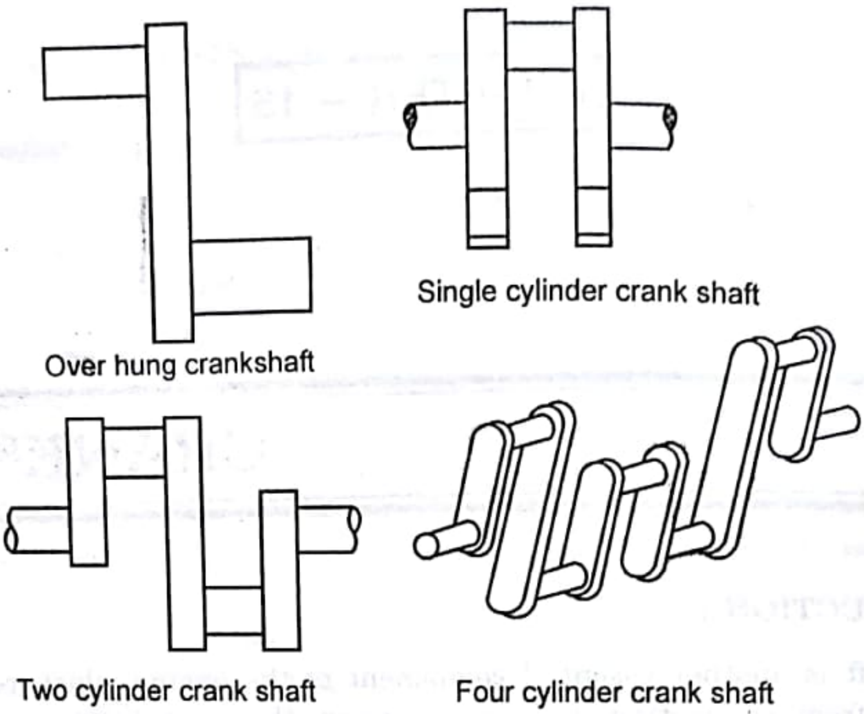

Fig. 15.1: Types of crank shafts

Overhung crank shaft has only one crank web whereas the centre crankshaft has many webs in such a way that single throw crankshaft consists of two webs and multi-throw crankshaft contains two webs for each crank pin. The various types of crankshaft are shown in figure 15.1.

15.3 MATERIALS FOR CRANKSHAFT :

Mostly crankshafts fail due to progressive fracture occured either from repeated bending or reversed torsional stresses. Hence the crankshaft should be tough and have high fatigue strength in order to safeguard the crankshaft from repeated bending and torsional loads. Also, it should be rigid enough to keep the distortion to a minimum.

Usually crankshafts are made of heat treated carbon steels such as C20 to C30 steel (i.e.; mild steel) and alloy steels of manganese, nickel and chromium. Carbon steel crankshafts are employed in industries and alloy steel crankshafts find applications in transportation vehicles such as automobiles including air-craft engines.

15.4 DESIGN OF OVERHUNG CRANKSHAFT : (Refer figure 15.2)

Overhung crankshaft or side crankshaft consists of one crank pin, one shaft part (i.e.; journal) and one web which connects the crank pin with the journal. When designing the crankshaft, it is required to discuss about the nature of stresses induced in various parts of the crankshaft.

Let

F = Force transmitted from connecting rod to the crankshaft.
A = Area of cross-section of crank pin.
l = Length of crank-pin.
d = Diameter of crank pin
L = Length of main journal (i.e.; shaft-part)
D = Diameter of main journal.
w = Width of crank-web.
t = Thickness of crank-web.
r = Distance between axes of crank-pin and journal (i.e.; crank radius)
x = Distance between the centres of crank-pin and journal.
θ = Angle of inclination of the crank from inner dead centre.
ϕ = Angle of inclination of the connecting rod with the line of stroke.
β = Angle between crank and connecting rod.
F_r = Radial component of force.
F_t = Tangential component of force.
(S_b) = Allowable bending stress.
(S_s) = Allowable shear stress.
(S_c) = Allowable crushing (or) bearing stress.

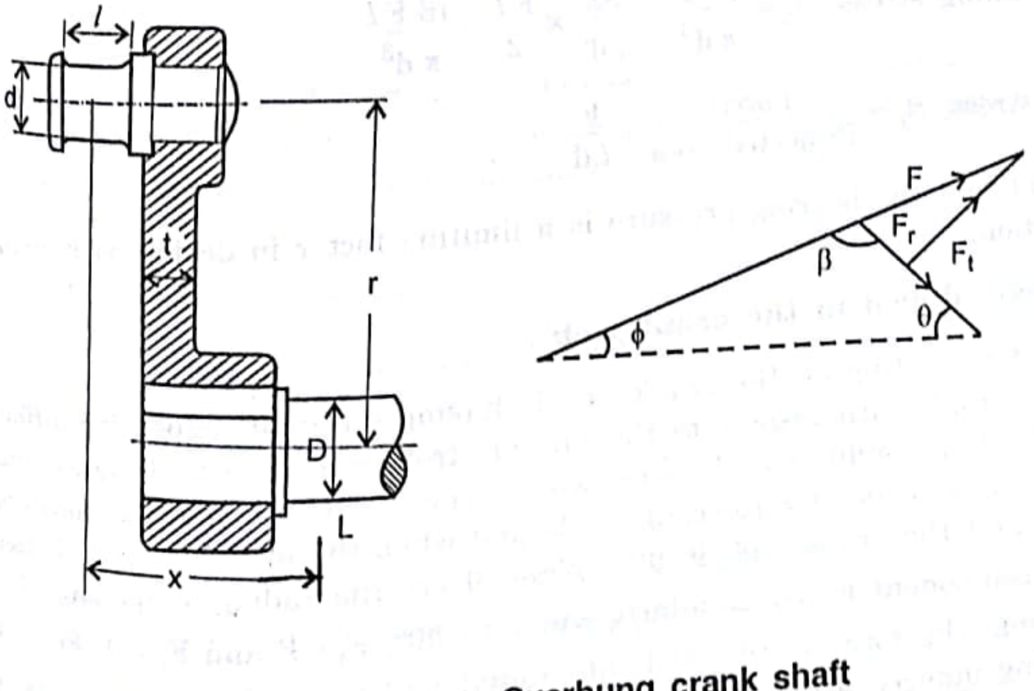

Fig. 15.2: Overhung crank shaft

15.4.1 Stresses induced in the crank-pin :

When the force is transmitted from the connecting rod to the crankshaft, the crank pin is subjected to three types of stresses namely,

(i) Plain shear stress due to direct shear force.

(ii) Bending stress at the fixed end due to the bending moment.

(iii) Crushing (or) bearing stress acting over the projected area.

At any crank angle θ, the force F can be resolved into radial component of force F_r, and tangential component of force F_t. Their magnitudes are

$$F_r = F\cos(\theta+\phi) \text{ and } F_t = F\sin(\theta+\phi)$$

In the case of crank-pin, these components of force will not produce any effect on the pin and hence, for the design of crank pin, the actual force F may be considered for all positions of the crank.

Now, the various stresses induced in the crank-pin are evaluated as follows.

Plain shear stress, $S_s = \frac{F}{A} = \frac{F}{(\pi/4)\,d^2} = \frac{4\,F}{\pi\,d^2}$

Bending moment at the fixed end, $M = F \times \frac{l}{2}$

(Assuming the force is acting at the centre of pin).

Hence bending stress. $S_b = \frac{32\,M}{\pi\,d^3} = \frac{32}{\pi\,d^3} \times \frac{F\,l}{2} = \frac{16\,F\,l}{\pi\,d^3}$

Bearing stress, $S_c = \frac{\text{Force}}{\text{Projected area}} = \frac{F}{l.d}$

It is found that the bearing pressure is a limiting factor in design as it insures proper lubrication.

15.4.2 Stresses induced in the crank-web :

Since the force acting on the crank-web is having different values for different positions of the crank with respect to the line of stroke, the web is designed based on maximum loading conditions. Usually two positions of crank may be considered for the web design, that is, at zero crank angle and when the included angle between connecting rod and the crank web is 90°. When $\theta = 0$, the radial component $F_r = F$ and tangential component $F_t = 0$. Similarly when $\beta = 90°$, $F_t = F$ and $F_r = 0$. For other positions of crank, the force is resolved into radial and tangential components and the corresponding induced stresses are evaluated properly.

15.6.3. Stresses induced in the crank shaft main journal :

Since the centre crankshaft is similar to simply supported beam, the bending moment at the journals is zero. Hence the possible induced stress is due to twisting moment produced by the tangential force.

i.e.; The torsional shear stress, $S_s = \frac{16T}{\pi D^3} = \frac{16 F_t \cdot r}{\pi D^3}$

where $F_t = \frac{F}{2} \sin(\theta + \phi)$

Sometimes, flywheels may be connected at the end of journal. For such cases, the bending moment produced by the weight of the flywheel on the journal may be taken into account for the design consideration.

15.6.4. Design of main bearings :

The main bearings, into which the crankshaft journals are rotating, are designed based on the bearing pressure developed over the projected area.

If D = Diameter of bearing.

L = Length of bearing.

then, bearing pressure, $p_b = \frac{\text{Load}}{\text{Projected area}} = \frac{W}{L.D}$

where, W = F for overhung crankshaft.

and $W = \frac{F}{2}$ for centre crankshaft.

15.7. DESIGN OF MULTI-THROW CRANK-SHAFT :

Since the multi-throw crankshaft is simply the multiple structure of single throw crankshaft, the design of multi-throw crankshaft is very similar to the design of single throw crank-shaft. In this case, since all the cylinders of the engine possess equal capacity, the force supplied by one cylinder is used for designing one portion of the crankshaft, (i.e.; one set of crank pin, web and journal etc) and for the remaining portions, the same design values are adopted.

15.8. DESIGN STRESS VALUES :

All parts of crank-shaft (i.e.; crank pin, web and journal) are made of same material and hence they must have common design stress values. Usual design stress values for the crankshaft material (i.e; for mild steel) are,

(i) In bending : 60 to 100 MPa

(ii) In tension & compression : 80 to 120 MPa

(iii) In shear : 40 to 60 MPa

(iv) In bearing : 10 to 20 MPa

The design bearing pressure for the bearings are

(i) In crank pin : 4 to 12 MPa

(ii) In main shaft : 1.5 to 12 MPa

15.9. STEPS INVOLVED IN THE DESIGN OF CRANKSHAFT :

1. From the given problem, identify the type of crankshaft to be designed, material, steam or gas pressure and other given parameters.

2. Determine the maximum load acting on the crank pin, maximum torque and bending moments.

3. Findout the parameters of crank pin such as its length, and diameter etc. based on the bearing pressure and check the induced bending and shear stresses with their allowable values.

4. Design the main journal (i.e; shaft) based on maximum torque and bending moment conditions and check the bearing pressure.

5. Select the web parameters proportionately and check their induced stresses.

6. In any case, if the induced stress is more than the allowable value, then alter the corresponding dimension suitably.

7. Usually, the following proportions are adopted for the crankshaft parts.

Let d = Diameter of crank pin.

D = Diameter of main journal.

Then for overhung crankshaft.

(a)	Diameter of main journal, D	= 1.25 to 1.5d
(b)	Length of main journal, L	= 1.25D
(c)	Length of journal inside the crank, L_1	= 1.0 to 1.25D
(d)	Length of crank pin, l	= 1.0 to 1.25d
(e)	Length of pin inside the crank, l_1	= 1.0 to 1.25d
(f)	Thickness of web, t	= 0.7 to 1.0d
(g)	Width of web nearer to crank pin, a	= 1.5d
(h)	Width of web nearer to journal, b	= 1.5D

For centre crankshaft,

(a) Diameter of journal, D = d

(b) Thickness of web, t = 0.7d

(c) Width of web, w = 1.5d

The remaining parameters may be calculated based on design stress values.

15.10. SOLVED PROBLEMS :

Problem 15.1 :

Design an overhung crank shaft for the following data.

Maximum load on the crank pin for maximum torque position = 50 KN

Crank radius = 200 mm

Distance between crank pin centre and near by bearing centre = 300 mm

Allowable stress in

bending	:	**70 MPa**
shear	:	**50 MPa**
bearing	:	**7 MPa**

(Madras University)

Solution : (Refer figure 15.4)

Given :

Maximum load at maximum torque position F = 50 KN = 50000 N

Crank radius r = 200 mm

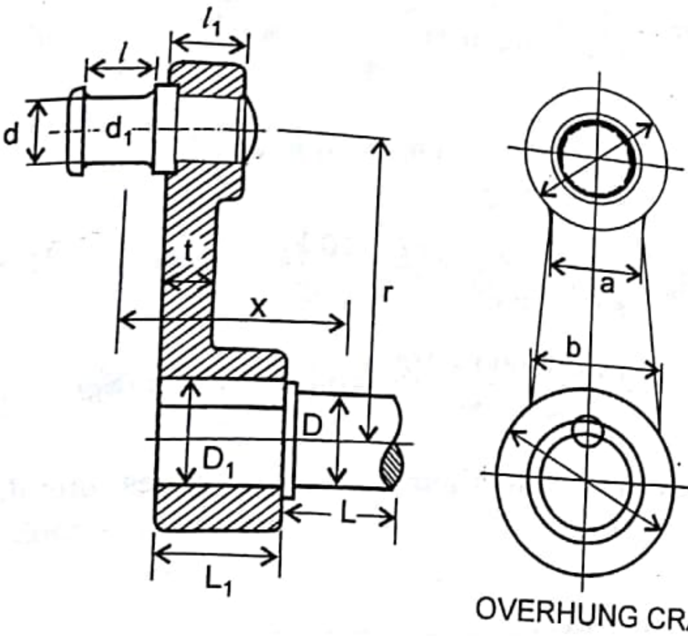

Fig. 15.4

Distance between pin and bearing, x = 300 mm.

Allowable bending stress, $(S_b) = 70\ MPa = 70 \times 10^6\ N/m^2 = 70\ N/mm^2$

Allowable shear stress, $(S_s) = 50\ MPa = 50 \times 10^6\ N/m^2 = 50\ N/mm^2$

Allowable bearing stress, $(S_c) = 7\ MPa = 7 \times 10^6\ N/m^2 = 7\ N/mm^2$

Design of crank pin :

Let l = Length of crank pin

d = Diameter of crank pin

Crank pin is designed based on bearing stress as

$$\text{Bearing stress, } S_c = \frac{\text{Load}}{\text{Projected area}} = \frac{F}{l.d}$$

$$\text{Assuming } l = 1.2d, \text{ we get } S_c = \frac{F}{1.2d \times d}$$

$$\text{(or) } d = \left[\frac{F}{1.2 \times S_c}\right]^{1/2} = \left[\frac{50000}{1.2 \times 7}\right]^{1/2} = 77mm = 80mm \text{ (say)}$$

$$\therefore l = 1.2d = 1.2 \times 80 = 96mm = 100mm \text{ (say)}$$

Now the induced shear and bending stresses are checked as follows.

Induced shear stress, $S_s = \frac{\text{Load}}{\text{Shearing area}} = \frac{F}{\frac{\pi}{4} d^2} = \frac{4F}{\pi d^2}$

$$= \frac{4 \times 50000}{\pi \times 80^2} = 9.95 \text{ N/mm}^2 < (S_s)$$

Induced bending stress, $S_b = \frac{32M}{\pi d^3} = \frac{32}{\pi d^3} \times \frac{Fl}{2} = \frac{16 Fl}{\pi d^3}$

$$= \frac{16 \times 50000 \times 100}{\pi \times 80^3} = 50 \text{ N/mm}^2 < (S_b)$$

Since the induced stresses are less than their allowable values, our design is safe.

Design of main journal :

Let D = Diameter of main journal

L = Length of main journal

Assume $D = 1.5d = 1.5 \times 80 = 120\text{mm}$

and $L = 1.5D = 1.5 \times 120 = 180\text{mm}$

Now the induced stress at maximum torque position is determined as follows.

Bending moment by tangential force, $M = F_t \cdot x = F \cdot x$

$$= 50000 \times 300 = 15 \times 10^6 \text{ N-mm.}$$

Twisting moment by the tangential force,

$$T = F_t \cdot r = F \cdot r = 50000 \times 200 = 10 \times 10^6 \text{ N-mm}$$

(Note :- Since the crank angle θ is not mentioned in the problem, we may assume that the maximum torque is obtained as $\beta = 90°$, i.e; when the angle between the crank and connecting rod is 90°. In this position, the radial force $F_r = 0$, and tangential component $F_t = F$).

Now the equivalent bending moment, $M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$

$$= \frac{1}{2}\left[(15 \times 10^6) + \sqrt{(15 \times 10^6)^2 + (10 \times 10^6)^2}\right]$$

$$= 16.5 \times 10^6 \text{ N-mm}$$

Equivalent bending stress, $S_b = \frac{32\ M_e}{\pi\ D^3}$

$$= \frac{32 \times 16.5 \times 10^6}{\pi \times 120^3} = 97.3\ N/mm^2 > (S_b)$$

Since the induced stress is more than allowable value, let us increase D = 140 mm.

Now $S_b = \frac{32 \times 16.5 \times 10^6}{\pi \times 140^3} = 61.2\ N/mm^2 < (S_b)$

Equivalent shear stress, $S_s = \frac{16\ T_e}{\pi\ D^3}$

where $T_e = \sqrt{M^2 + T^2} = \sqrt{(15 \times 10^6)^2 + (10 \times 10^6)^2} = 18 \times 10^6\ N - mm.$

Now $S_s = \frac{16 \times 18 \times 10^6}{\pi \times 140^3} = 33.4\ N/mm^2 < (S_s)$

Bearing stress, $S_c = \frac{F}{L.D}$

Assuming $L = 1.5D = 1.5 \times 140 = 200$ mm, we get

$$S_c = \frac{50000}{200 \times 140} = 1.8\ N/mm^2 < (S_c)$$

Since all the induced stresses in the main journal are less than the allowable values, design of main journal is correct.

Design of web :

Let w = Width of web

t = Thickness of web.

Now $t = 300 - \left(\frac{l}{2} + \frac{L}{2}\right) = 300 - (50 + 100) = 150$ mm

[∵ the distance between the pin centre and journal centre is 300 mm (given)]

Width of web nearer to pin, $a = 1.5\ d = 1.5 \times 80 = 120$ mm.

Width of web nearer to journal, $b = 1.5\,D = 1.5 \times 140 = 210$ mm.

For design analysis, the average width may be considered, which is

$$w = \frac{a+b}{2} = \frac{120+210}{2} = 165 \text{ mm.}$$

The induced stresses are,

Direct stress, $S_0 = \frac{F}{w.t} = \frac{50000}{165 \times 150} = 2 \text{ N/mm}^2$

Bending stress due to tangential component $S_b = \frac{F_t \cdot r}{(1/6)\, t\, w^2} = \frac{6\, F \cdot r}{t\, w^2}$

$$= \frac{6 \times 50000 \times 200}{150 \times 165^2} = 15 \text{ N/mm}^2$$

Total stress, $S = S_0 + S_b = 2 + 15 = 17 \text{ N/mm}^2 < (S_b)$

Now also, our design is safe.

Now, the other parameters are assumed as,

Length of crank pin inside the web, $l_1 = t = 150$ mm

Diameter of crank pin inside the web, $d_1 = d = 80$ mm.

Length of main journal inside the web, $L_1 = t = 150$ mm

Diameter of main journal inside the web, $D_1 = D = 140$ mm.

Specifications :

1. Length of crank pin = 100 mm.
2. Diameter of crank pin = 80 mm
3. Length of journal = 200 mm
4. Diameter of journal = 140 mm
5. Width of web = 165 mm
6. Thickness of web = 150 mm
7. Crank radius = 200 mm