Code No: 113AH

R13

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, May/June - 2015

MATHEMATICS – III

(Common to EEE, ECE, EIE, AGE)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A **(25 Marks)**

1.a) Define an ordinary point of a differential equation. [2M]

b) Find the singular points of the differential equation.
$x^2(x+1)^2 y''+(x^2-1)y'+2y=0$ [3M]

c) Write Rodrigue's formula and use it to write $P_1(x)$ as a polynomial. [2M]

d) Given that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ *and* $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$

Prove that $J_{-3/2}(x)=-\sqrt{\frac{2}{\pi x}}\left(\sin x+\frac{\cos x}{x}\right)$ [3M]

e) Find the value of a if cosax sinhy is harmonic. [2M]

f) Evaluate $\int_C \frac{zdz}{(z+3)}, |z|=1.5.$ [3M]

g) Find the residue of $\frac{e^{iz}}{(z^2+1)}$ at z = i. [2M]

h) Expand $\frac{1}{z-1}$ when $|z|>1$. [3M]

i) Find the critical points of $w=z^2$. [2M]

j) Find the fixed points of $\frac{2z+1}{z+2}$. [3M]

PART-B **(50 Marks)**

2. Solve in series the equation $xy''+2y'+xy=0$. [10]

OR

3. Solve in series the equation $y''+xy'+(x^2+2)y=0$. [10]

4.a) Prove that $e^{\frac{1}{2}x(t-1/t)}=\sum_{n=-\infty}^{\infty} t^n J_n(x)$

b) Show that $P_{2n}(0)=\frac{(-1)^n(2n)!}{2^{2n}(n!)^2}$. [5+5]

OR

5.a) Prove that $P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$

b) Show that $J_0^2+2J_1^2+2J_2^2+2J_3^2+.....=1$ [5+5]

6.a) If $F(\xi)=\oint_C \frac{4z^2+z+5}{z-\xi}dz$, where C is the ellipse $(x/2)^2+(y/3)^2=1$, find the value of (i) $F(3.5)$ (ii) $F(i)$.

b) If $f(z)$ is an analytic function of Z, prove that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2+\left\{\frac{\partial}{\partial y}|f(z)|\right\}^2=|f'(z)|^2$$ [5+5]

OR

7.a) Evaluate $\int_C \bar{z}\,dz$ where C is (i) the line segment joining the points (1,1) and (2,4) (ii) The curve $x=t,\ \ y=t^2$ joining the points (1, 1) and (2, 4).

b) Evaluate $\oint_C \frac{e^z}{(z^2+\pi^2)^2}dz,\ where\ \ C\ is\ |z|=4.$ [5+5]

8. Expand $f(z)=\frac{1}{[(z-1)(z-2)]}$ in the region:

a) $|z|<1$, b) $1<|z|<2$ c) $|z|>2$. [10]

OR

9. Using contour integration evaluate, $\int_0^{\pi}\frac{a\,d\theta}{a^2+\sin^2\theta},\ a>0$. [10]

10.a) Find the image of the region $y>1$ under the transformation $w=iz+1$.

b) Find the image of the strip $2<x<3$ under $w=1/z$. [5+5]

OR

11.a) Show that the transformation $w=\frac{z-1}{z+1}$ maps the imaginary axis x=0 onto the circle $|w|=1$. What portion of the z-plane corresponds to the interior of the circle $|w|=1$.

b) Find the bilinear transformation which maps $-1,0,1$ of the z-plane onto $-1,-i,\ 1$ of the w-plane. [5+5]

---ooOoo---