1.224J/ ESD.204J: Airline Crew Scheduling

Outline

- Crew Scheduling
  - Problem Definition
    - Costs
  - Set Partitioning Model and Solution
- Enhanced Crew Scheduling
  - Link to Maintenance Routing Problem
  - Enhanced Model and Solution Approach

Airline Schedule Planning

Schedule Design ↓	Select optimal set of *flight legs* in a schedule
Fleet Assignment ↓	A flight specifies origin, destination, and departure time
Aircraft Routing ↓	Route individual aircraft honoring maintenance restrictions
Crew Scheduling	Assign crew (pilots and/or flight attendants) to flight legs

Crew Scheduling: Some Background

- This problem has been studied by operations researchers for at least 4 decades
- Most major U.S. airlines use crew pairing optimizers for the cockpit crews
  - Crew costs are the airlines' second largest operating expense
  - Even small improvements in efficiency can have large financial benefits

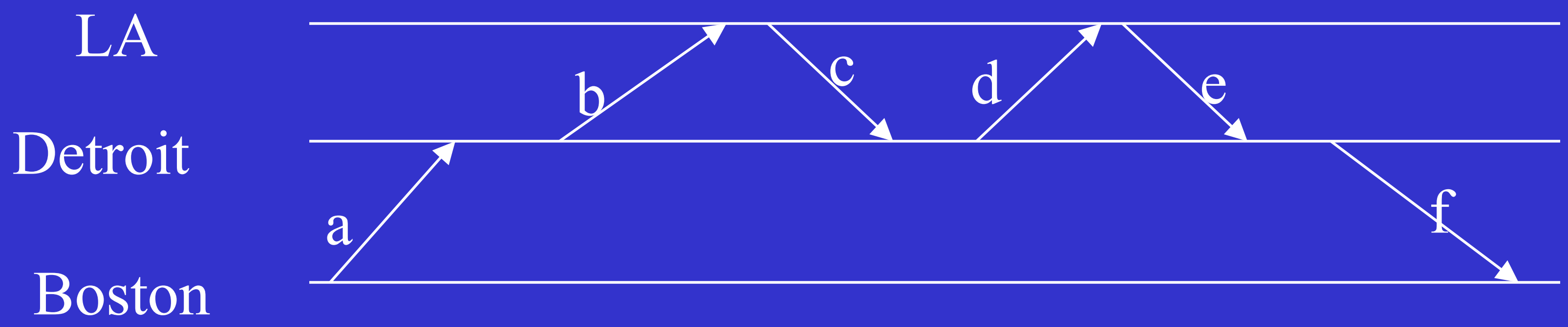
Airline Crew Scheduling

- 2-stage process:
  - Crew Pairing Optimization
    - Construct minimum cost work schedules, called pairings, spanning several days
  - Bidline Generation/ Rostering
    - Construct monthly work schedules from the pairings generated in the first stage
      - Bidlines
      - Individualized schedules
    - Objective to balance workload, maximize number of crew requests granted, etc.

Some Definitions

- **A *crewbase* is the home station, or domicile, of a crew**
- **A *crew pairing* is a sequence of *flights* that can be flown by a single crew:**
  - beginning and ending at a crewbase
  - spanning one or more days
  - satisfying FAA rules and collective bargaining agreements, such as:
    - maximum flying time in a day
    - minimum rest requirements
    - minimum connection time between two flights

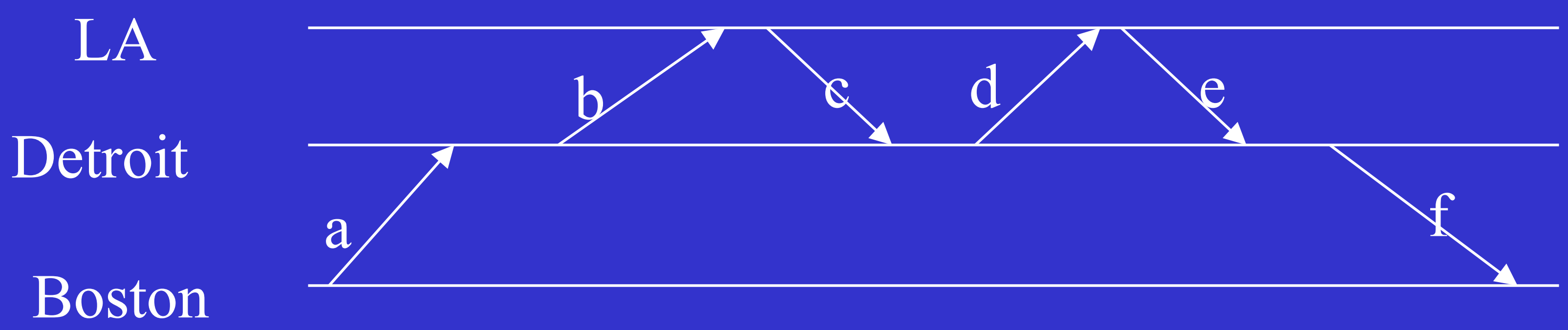
Example: A Crew Pairing



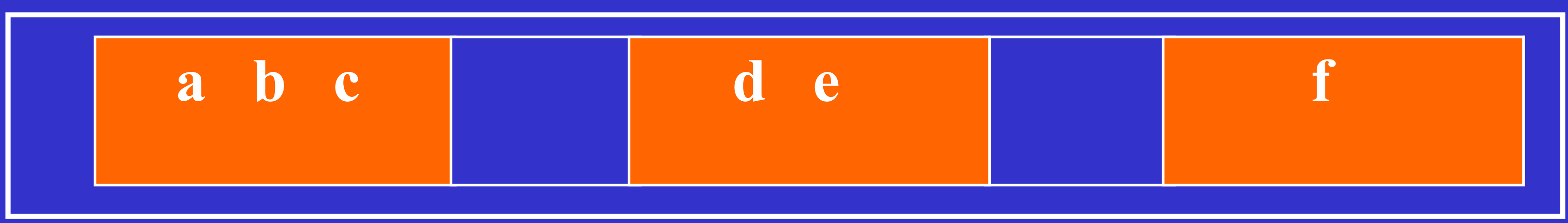
Some More Definitions

- **A *duty period* (or *duty*) is a sequence of flight legs comprising a day of work for a crew**
  - **Alternative pairing definition: a *crew pairing* is a sequence of *duties* separated by rests**
- **A *crew schedule* is a sequence of *pairings* separated by time-off, satisfying numerous restrictions from regulatory agencies and collective bargaining agreements**

Example: Duty Periods



Pairing = DP1(a,b,c) + rest + DP2(d,e) + rest + DP3(f)



Crew Pairing Problem (CP)

- Assign crews to flights such that every flight is covered, costs are minimized and labor rules are satisfied:
  - Maximum flying time in a day
  - Minimum rest requirements
  - Minimum connection time

Crew Pairing Costs

- Duty costs: *Maximum* of 3 elements:
  - *f1**flying time cost
  - *f2**elapsed time cost
  - *f3**minimum guarantee

- Pairing costs: *Maximum* of 3 elements:
  - *f1**duty cost
  - *f2**time-away-from-base
  - *f3**minimum guarantee

Set Partitioning Model for CP: Variable Definition and Constraints

- A variable is a *pairing*
  - Binary variables: =1 if pairing is assigned to a crew; = 0 if pairing not flown
- Set partitioning constraints require each flight to be covered exactly once
- Number of possible pairings (variables) grows exponentially with the number of flights

An Example

Flights:

A B C D E F G H

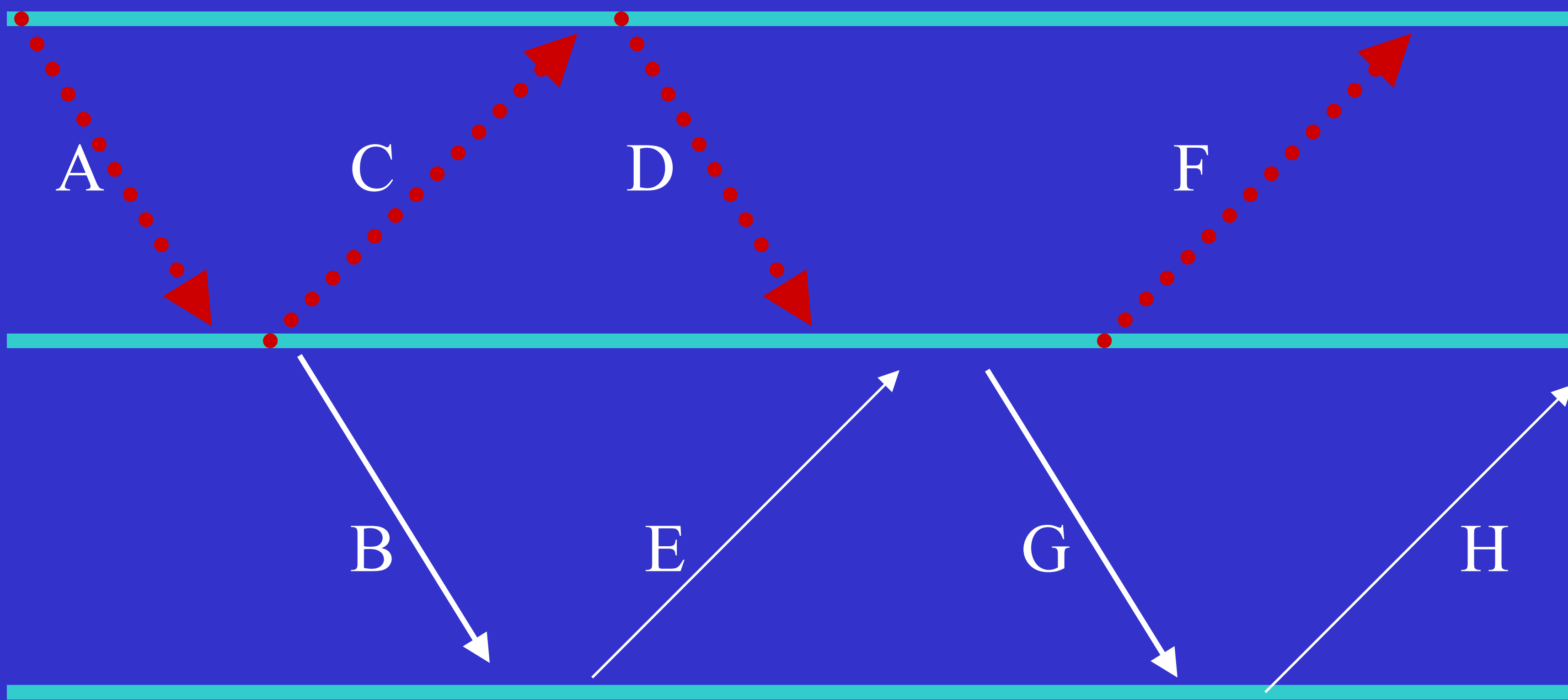
Potential pairings:

- A-C-D-F (y_1): \$1
- A-B-E-F (y_2): \$2
- C-D-G-H (y_3): \$4
- B-E-G-H (y_4): \$6

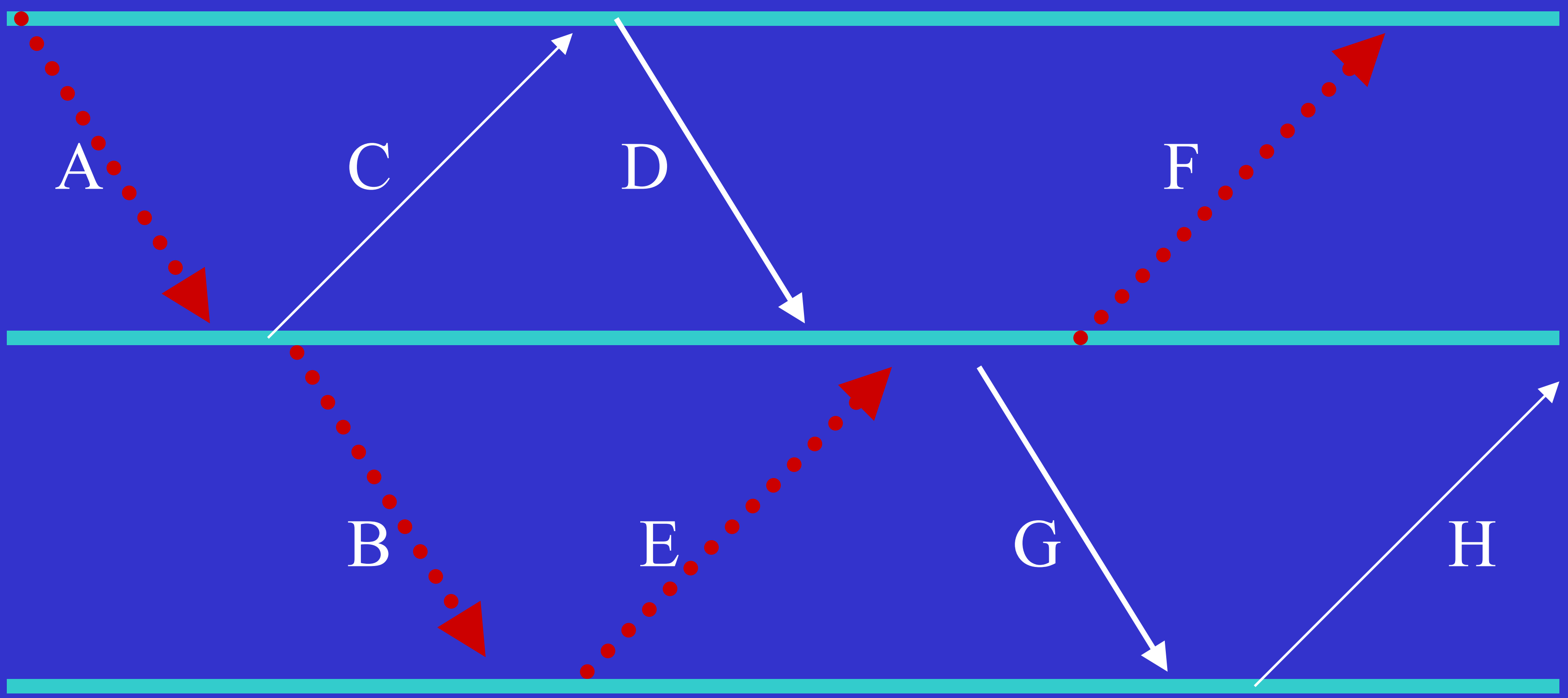
Crew pairing solutions:

- x_1 => pairings 1, 4: \$7
- x_2 => pairings 2, 3: \$6

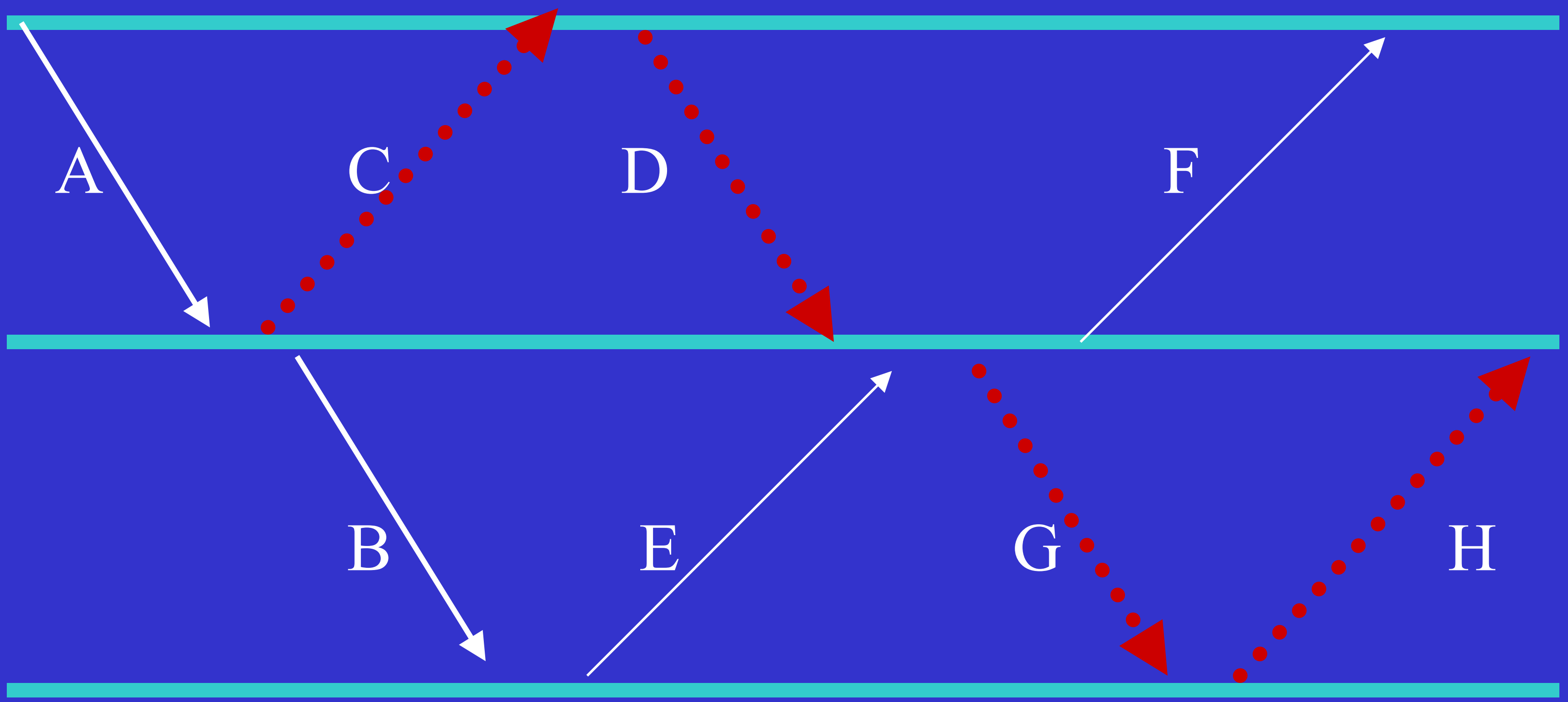
Pairing 1



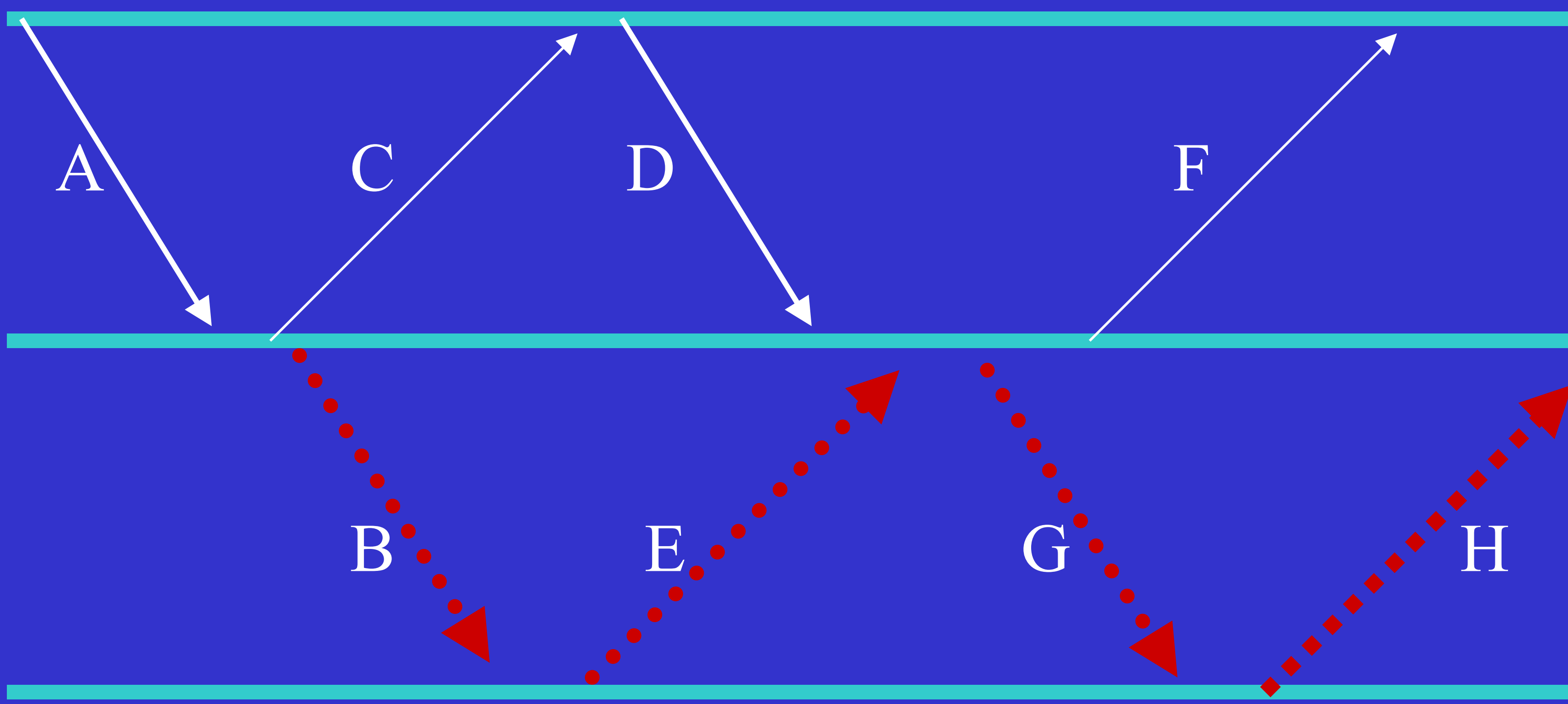
Pairing 2



Pairing 3



Pairing 4



Notation

- P^k is the set of feasible pairings for fleet family k
- F^k is the set of daily flights assigned to fleet family k
- δ_{fp} equals 1 if flight f is in pairing p, else 0
- c_p is the cost of pairing p
- y_p is 1 if pairing p is in the solution, else 0

Formulation

$$\min \sum_{p \in P^k} c_p y_p$$

$$st$$

$$\sum_{p \in P^k} \delta_{fp} y_p = 1 \quad \forall f \in F^k$$

$$y_p \in \{0,1\} \qquad \forall p \in P^k$$

Example

Flight Cover Constraints:

$$
\begin{array}{cccccccccc}
y_1 & + & y_2 & & & & & = & 1 & \text{A} \\
& & y_2 & + & & & y_4 & = & 1 & \text{B} \\
y_1 & + & & & y_3 & & & = & 1 & \text{C} \\
y_1 & + & & & y_3 & & & = & 1 & \text{D} \\
& & y_2 & + & & & y_4 & = & 1 & \text{E} \\
y_1 & + & y_2 & & & & & = & 1 & \text{F} \\
& & & & y_3 & + & y_4 & = & 1 & \text{G} \\
& & & & y_3 & + & y_4 & = & 1 & \text{H}
\end{array}
$$

Binary Pairing Restrictions:

$$
\begin{array}{cccccccccc}
y_1 & & & & & & & \in & 0,1 & 1 \\
& & y_2 & & & & & \in & 0,1 & 2 \\
& & & & y_3 & & & \in & 0,1 & 3 \\
& & & & & & y_4 & \in & 0,1 & 4
\end{array}
$$

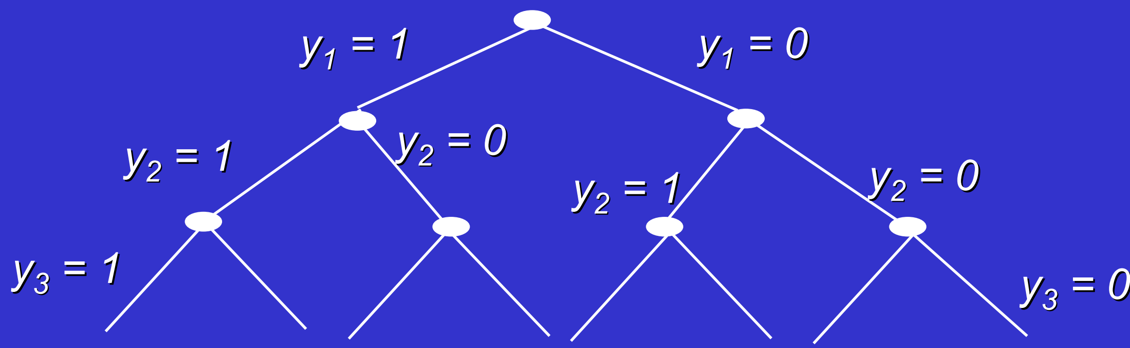
Set Partitioning Model: Advantages and Disadvantages

- Advantages:
  - Easy to model complex work rules
    - Very few constraints
    - Linear objective function and constraints
- Disadvantages:
  - Huge number of variables- number of variables grows exponentially with the number of flights

Problem Size

- A typical US airline (with a hub-and-spoke network) has millions or billions of potential pairings
  - Example
    - 150 flights 90,000 pairings
    - 250 flights 6,000,000 pairings
- Need a specialized approach to consider problems of this size

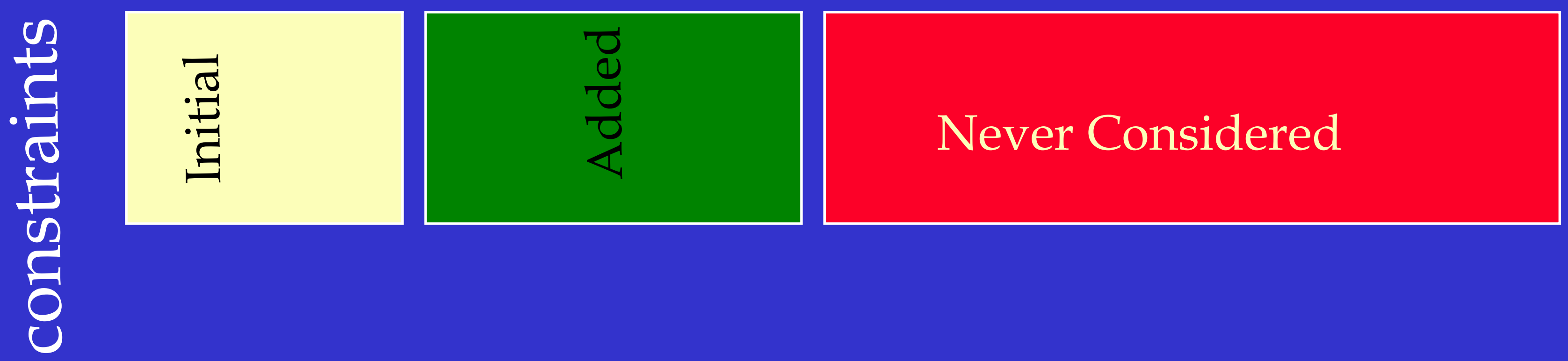
Branch-and-Price: Branch-and-Bound for Large-Scale Integer Programs



All possible solutions at leaf nodes of tree (2^n solutions, where n is the number of variables)

Column Generation

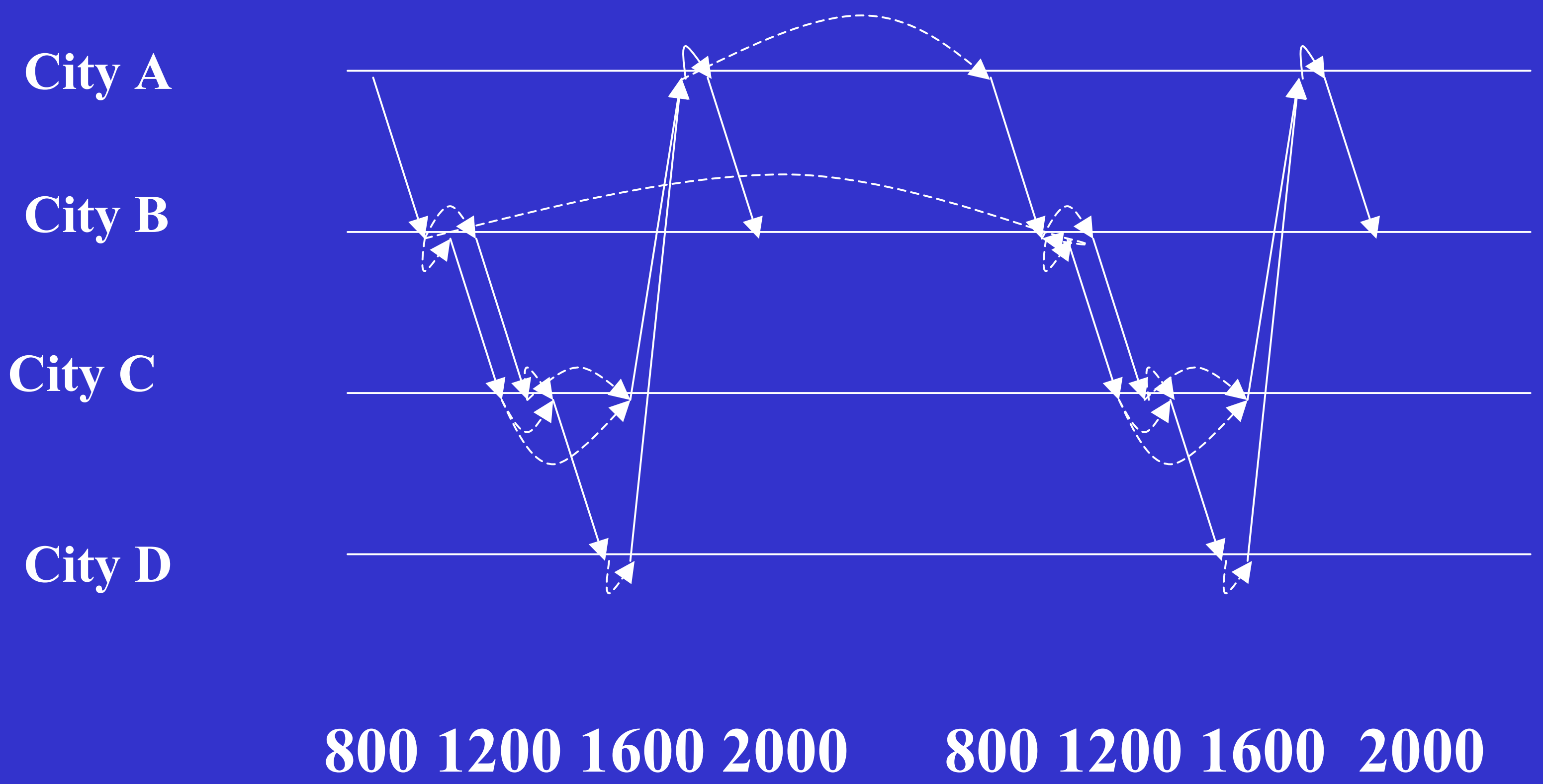
Millions/Billions of Variables



LP Solution: Column Generation

- Step 1: Solve Restricted Master Problem
- Step 2: Solve Pricing Problem (generate columns with negative reduced cost)
- Step 3: If columns generated, return to Step 1; otherwise STOP

Network Representation



Branch-and-Bound with Too Many Variables

- Branch-and-Price
  - Branch-and-bound with bounding provided by LP solutions
  - CP has too many variables to consider all of them
    - Solve linear programming relaxation using column generation

A Twist…

- Crew scheduling is critical to the airline industry
  - Second largest operating expense
  - Small improvement in solution quality has significant financial impact
- For decades, researchers have worked on finding better crew scheduling *algorithms*
- Our approach is to instead improve solution quality by *expanding the feasible set of solutions*

Airline Schedule Planning

Schedule Design

↓

Fleet Assignment

↓

Aircraft Routing

↓

Crew Scheduling

Route individual aircraft honoring maintenance restrictions

Assign crew (pilots and/or flight attendants) to flight legs

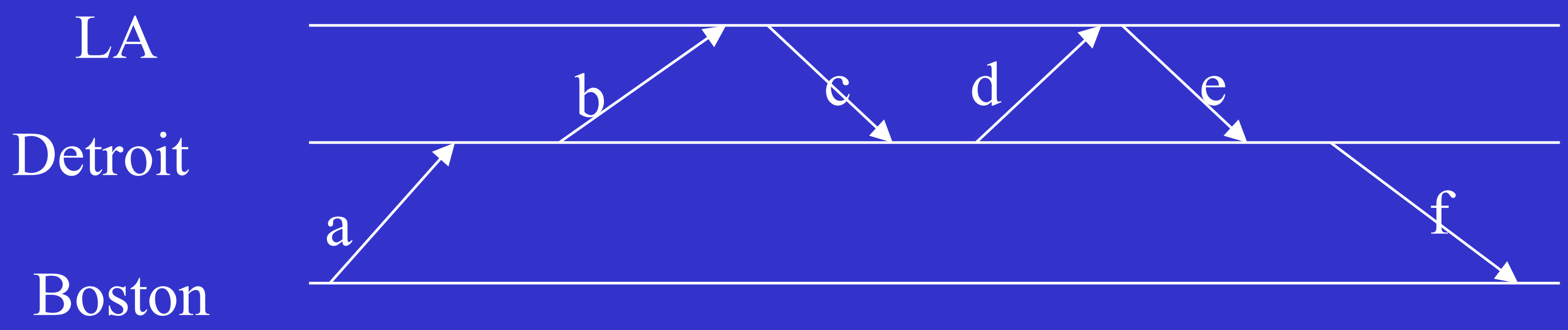
Aircraft Maintenance Routing: Problem Definition

- Given:
  - Flight Schedule for a single fleet
    - Each flight covered exactly once by fleet
  - Number of Aircraft by Equipment Type
    - Can't assign more aircraft than are available
  - Turn Times at each Station
  - FAA Maintenance Requirements
    - Maintenance required every 60 hours of flying
    - Airlines maintain aircraft every 40-45 hours of flying with the maximum time between checks restricted to three to four calendar days

Aircraft Maintenance Routing: Objective

- Find:
  - Feasible assignment of individual aircraft to scheduled flights
    - Each flight is covered exactly once
    - Maintenance requirements are satisfied
    - Conservation of flow (balance) of aircraft is achieved
    - The number of aircraft used does not exceed the number available

Example: Maintenance Station in Boston



String Model: Variable Definition

- A string is a sequence of flights beginning and ending at a maintenance station with maintenance following the last flight in the sequence
  - Departure time of the string is the departure time of the first flight in the sequence
  - Arrival time of the string is the arrival time of the last flight in the sequence + maintenance time

String Model: Constraints

- Maintenance constraints
  - Satisfied by variable definition
- Cover constraints
  - Each flight must be assigned to exactly one string
- Balance constraints
  - Needed only at maintenance stations
- Fleet size constraints
  - The number of assigned aircraft cannot exceed the number of aircraft in the fleet

Model Solution

- Complex constraints can be handled easily
- Model size
  - Huge number of variables
- Solution approach: branch-and-price
  - Generate string variables only as-needed

Airline Schedule Planning

Schedule Design

↓

Fleet Assignment

↓

Aircraft Routing

↓

Crew Scheduling

Route individual aircraft honoring maintenance restrictions

Assign crew (pilots and/or flight attendants) to flight legs

Integrate?

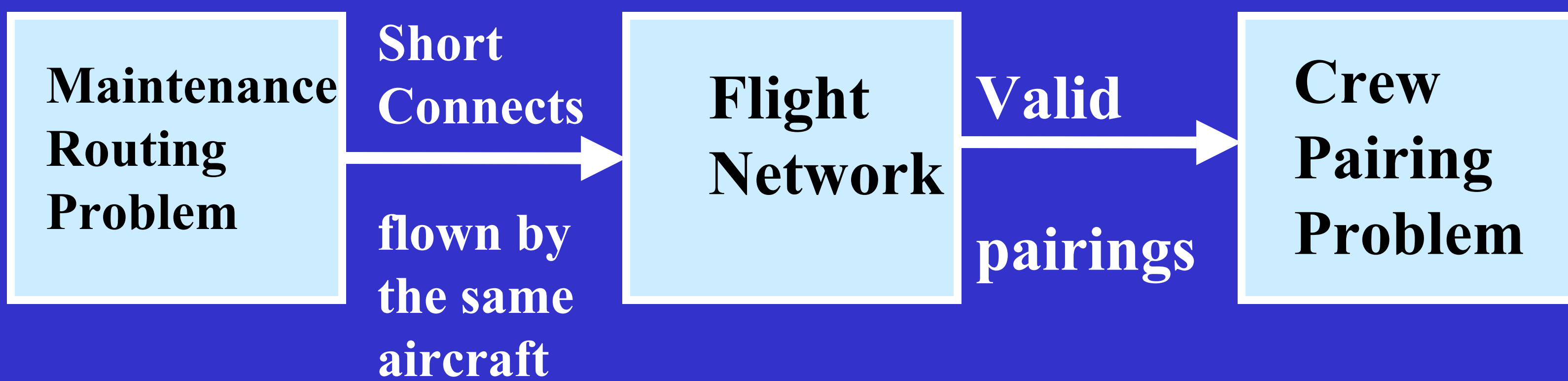
- Crew scheduling options are limited by maintenance routing decisions made earlier in the airline planning process
- Solving maintenance routing and crew scheduling simultaneously yields a large and challenging problem

➢ Idea is to improve crew scheduling by incorporating relevant maintenance routing decisions

Maintenance Routing and its Link to the Crew Pairing Problem

- The Maintenance Routing Problem (MR) - find ***feasible*** routing of aircraft ensuring adequate aircraft maintenance opportunities and flight coverage
- Crews need enough time between two sequential flights to travel through the terminal -- ***minimum connect time***
- If both flights are covered by the same aircraft, connection time can be reduced
- A ***short connect*** is a connection that is crew-feasible only if both flights are assigned to the same aircraft

Sequential Approach



Klabjan, Johnson, and Nemhauser

- Solve the crew pairing problem first, including all short connects in the crew pairing network
- Given the crew solution, require all short connects included in it to be part of the maintenance solution, which is solved second
- For “good” instances, this yields the optimal solution to the integrated problem (and many problems are “good”)
- For “bad” instances, this leads to maintenance infeasibility

Cordeau, Stojković, Soumis, Desrosiers

- Directly integrate crew and maintenance routing models
- Basic maintenance routing and crew pairing variables and constraints, plus linking constraints
- Benders decomposition approach using a heuristic branching strategy
- For non-hub-and-spoke networks, positive computational results

Our Approach

- Generate different solutions to the maintenance routing problem
- Allow the crew pairing model to choose the maintenance routing solution with the most useful set of short connects

The Example Again

Flights:

A B C D E F G H

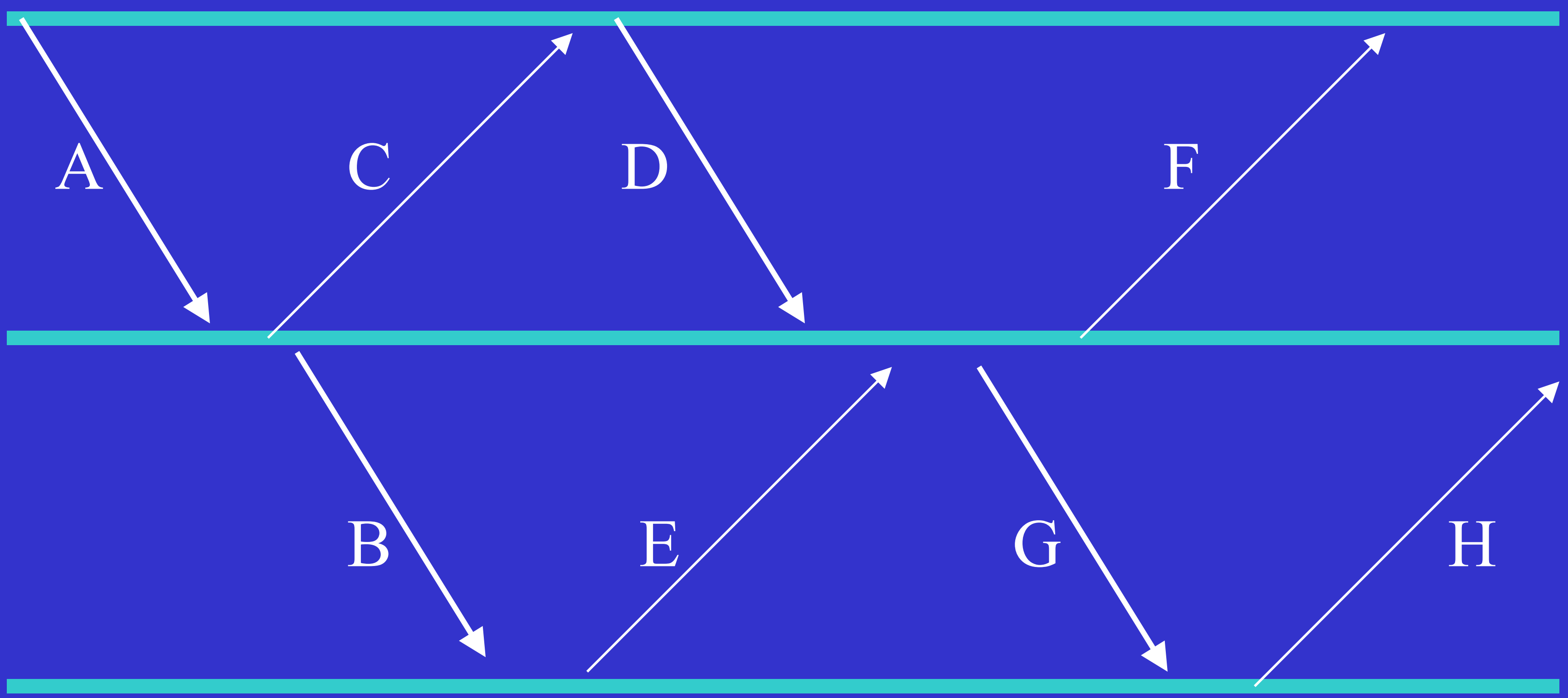
All Possible Short Connects:

A-B   A-C   E-G

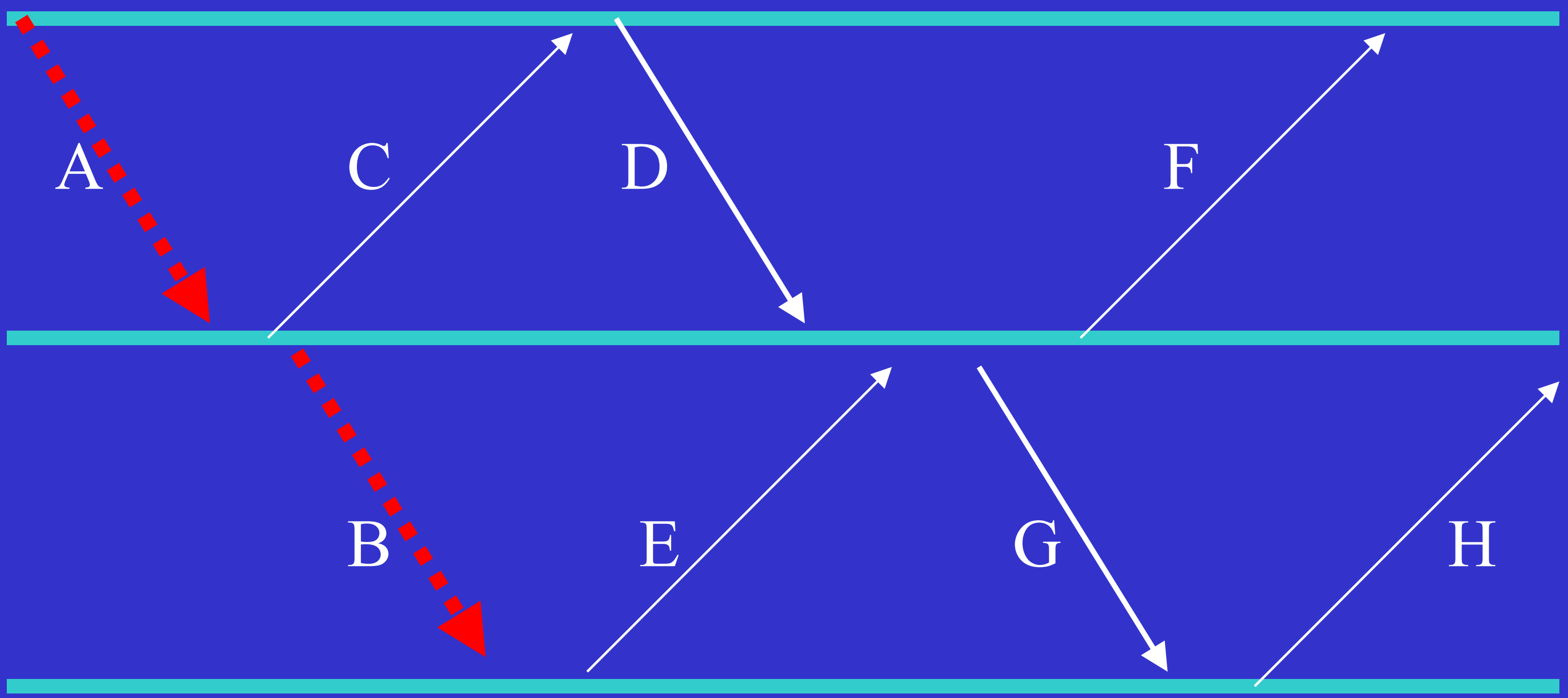
- MR solution (x_1) assigns the same aircraft to short connects A-C and E-G
- MR solution (x_2) assigns the same aircraft to short connect A-B

- Potential pairings:
  - A-C-D-F   (y_1): \$1
  - A-B-E-F   (y_2): \$2
  - C-D-G-H   (y_3): \$4
  - B-E-G-H   (y_4): \$6
- Crew pairing solutions:
  - x_1 => pairings 1, 4: \$7
  - x_2 => pairings 2, 3: \$6

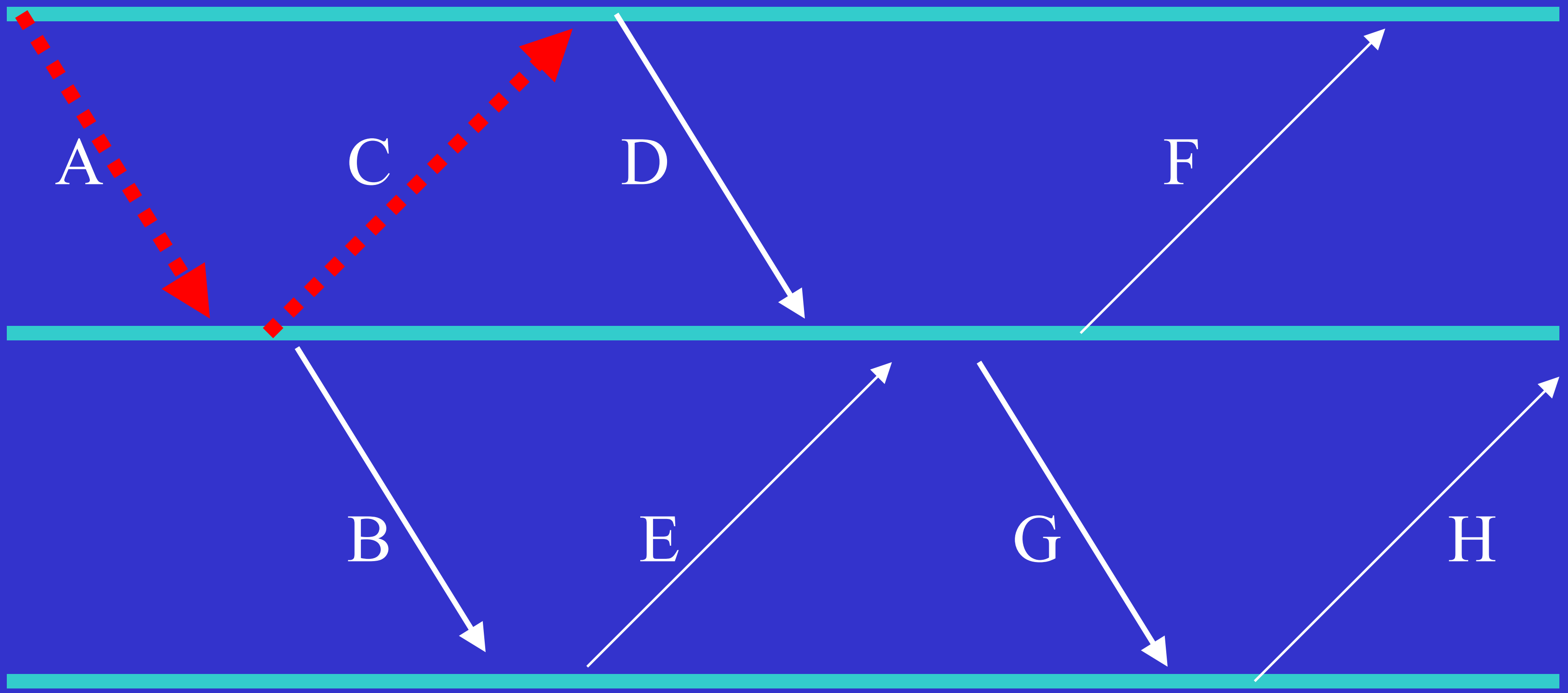
Flights



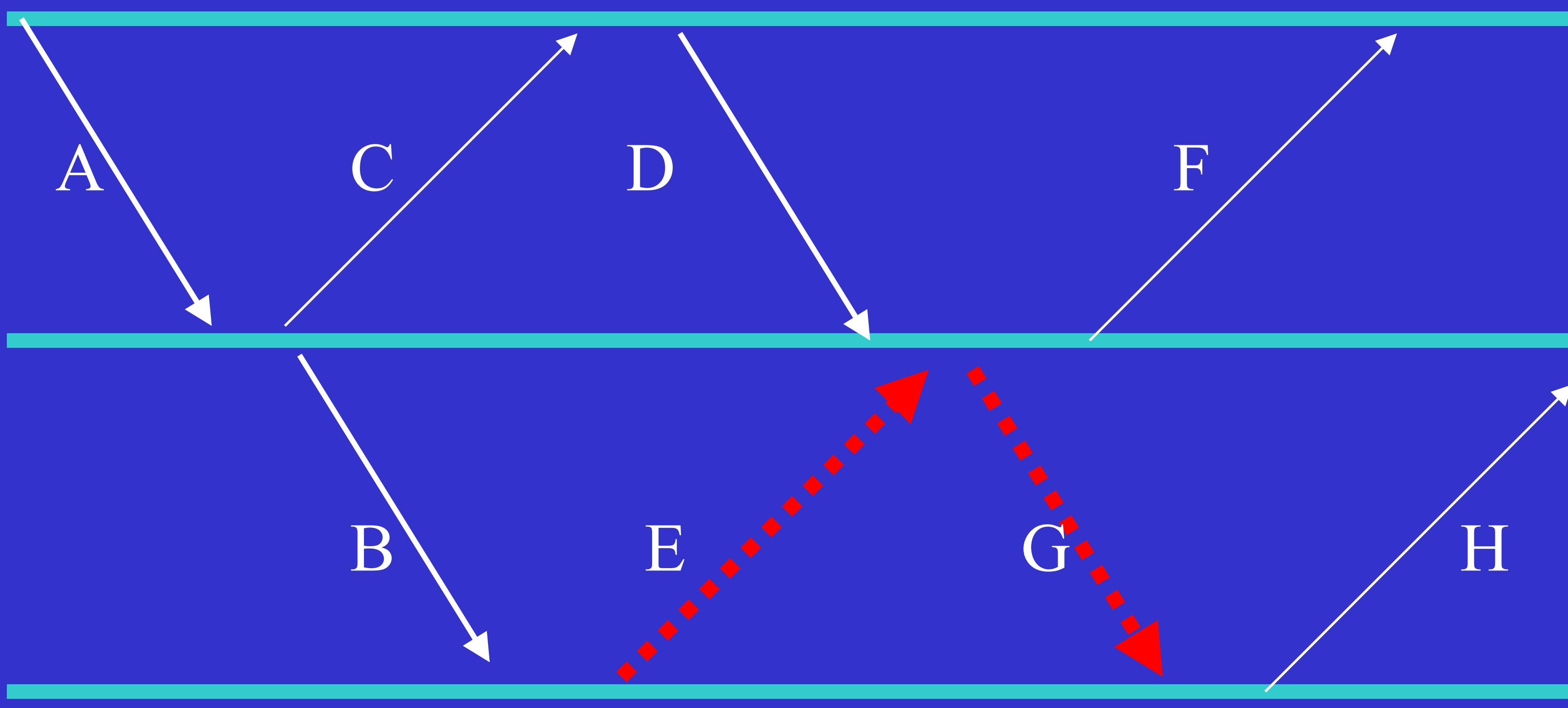
Short Connect



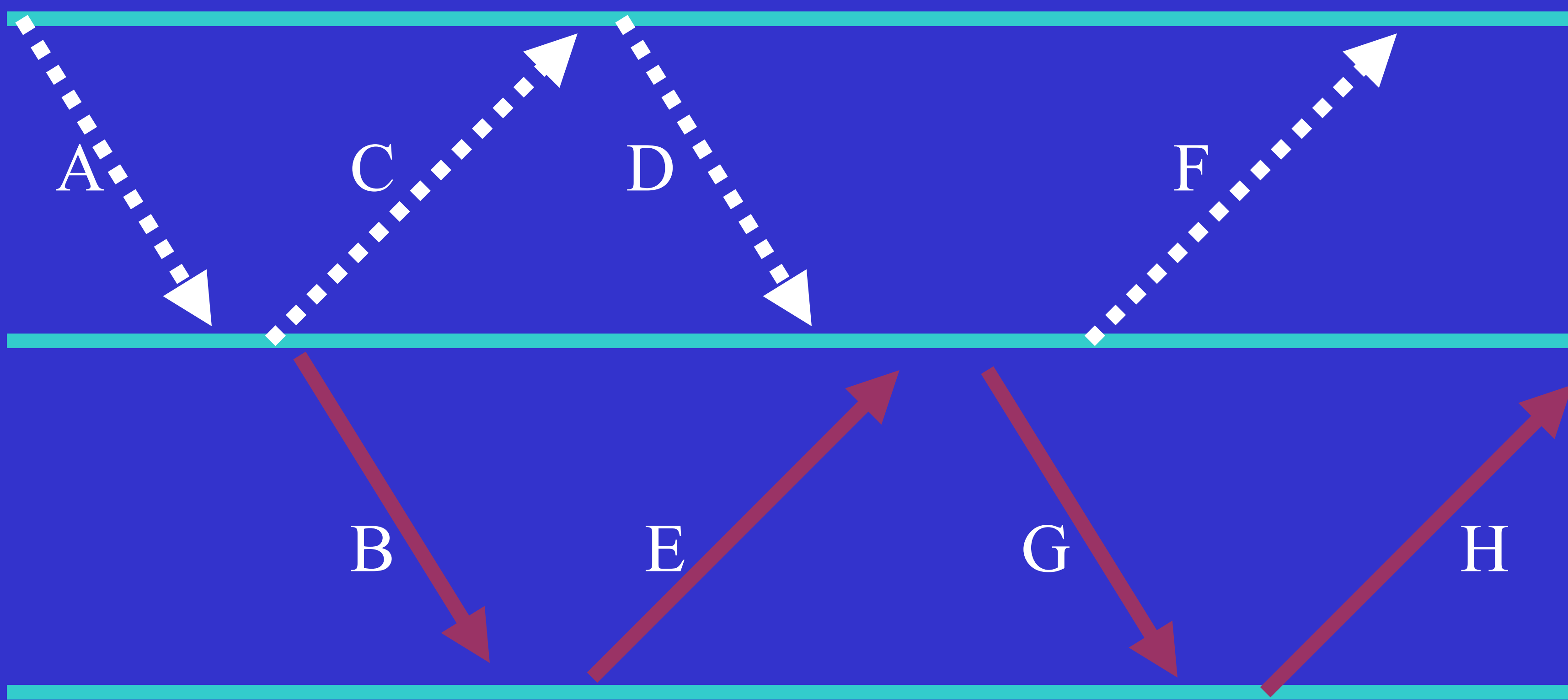
Short Connect



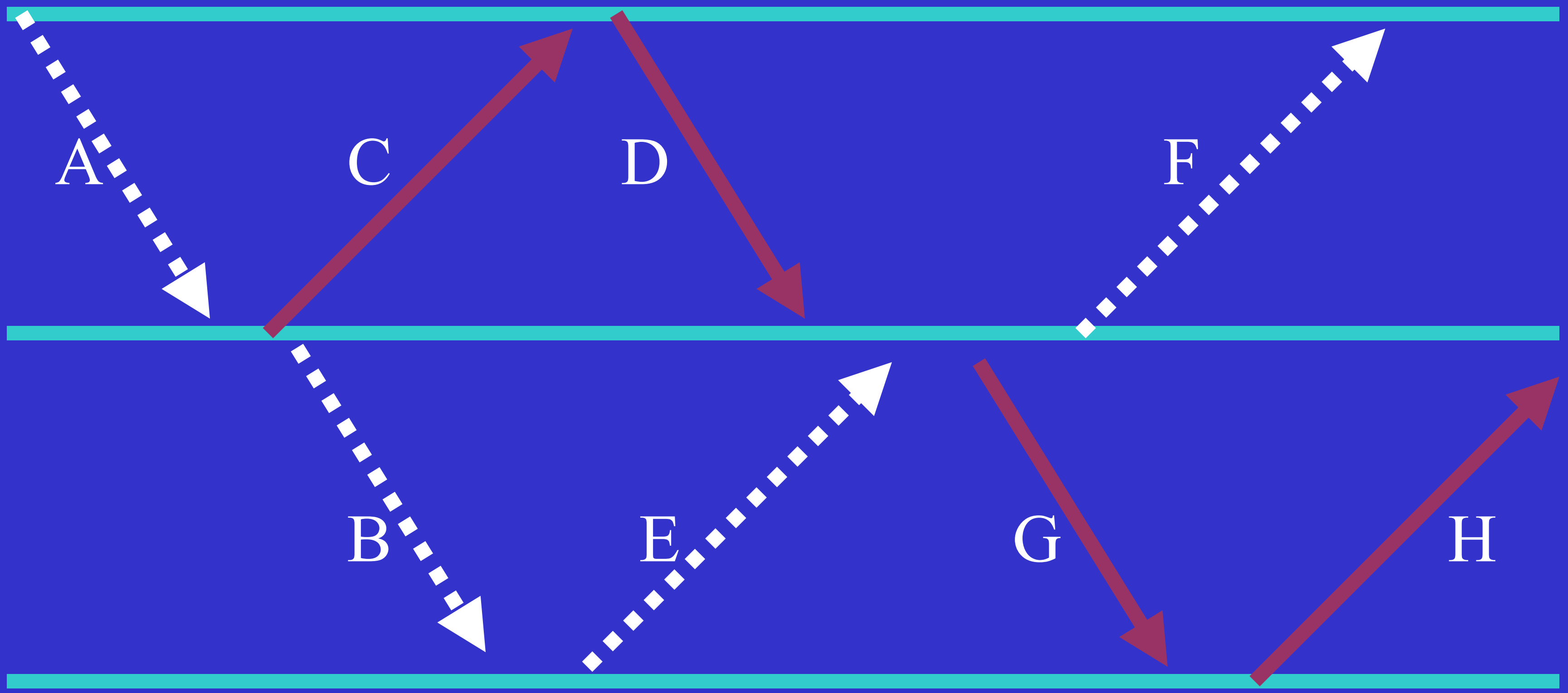
Short Connect



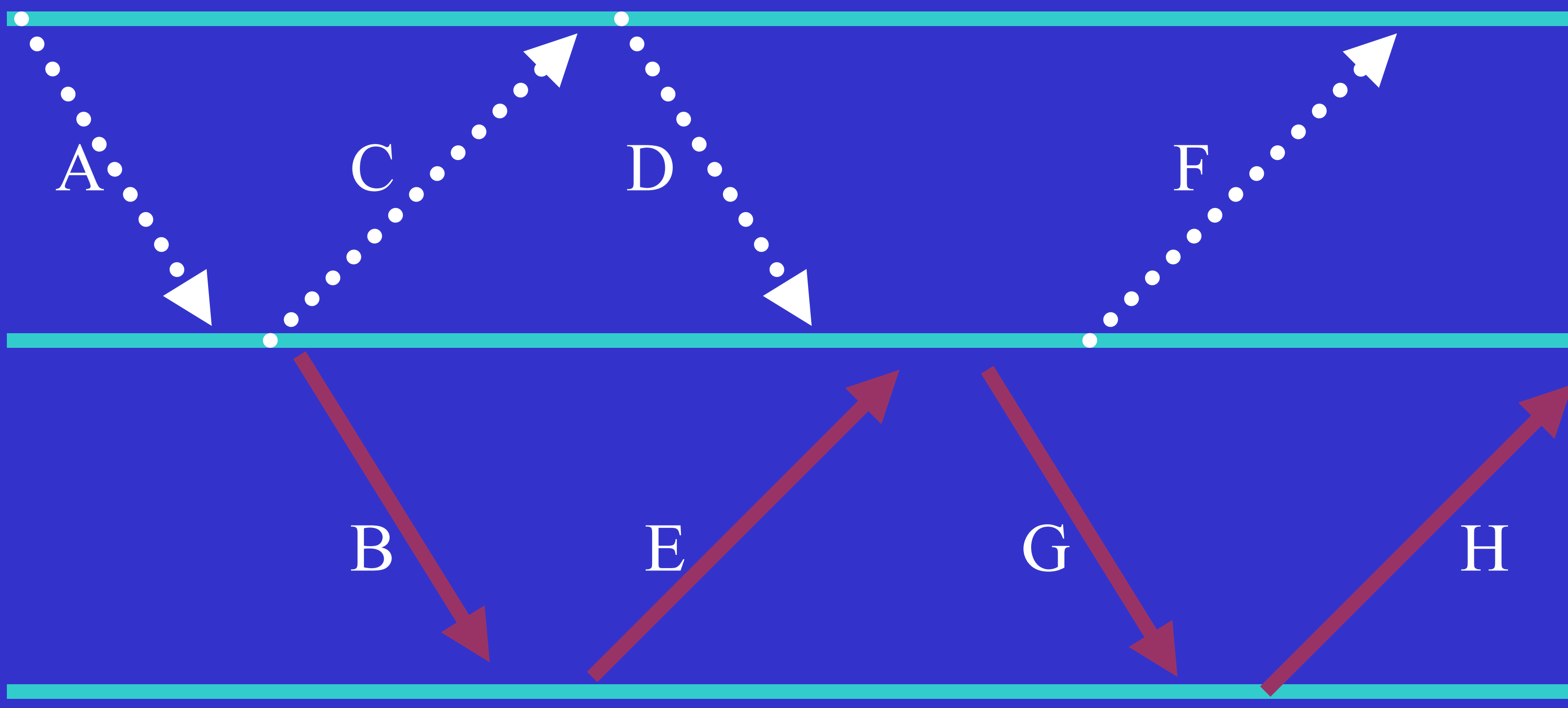
Maintenance Solution 1



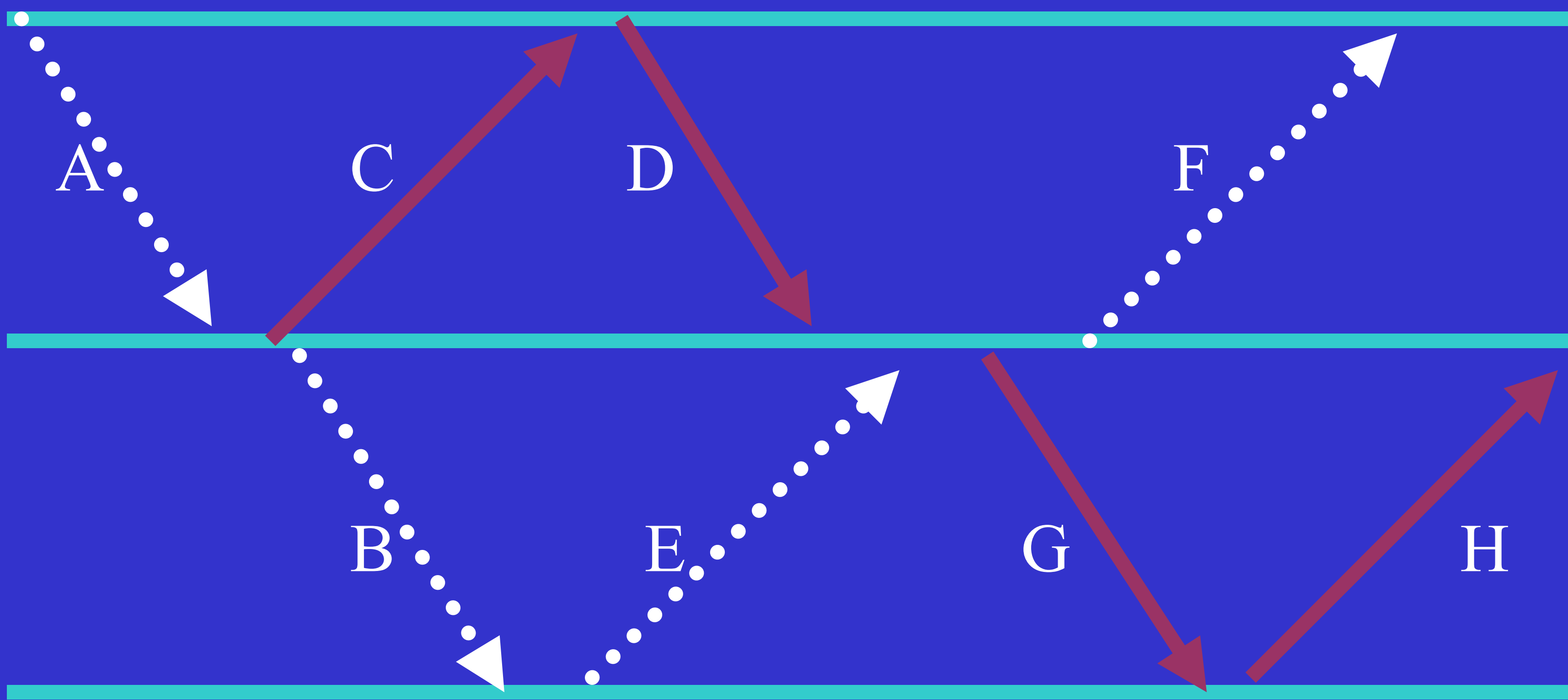
Maintenance Solution 2



If MR Solution 1 (A-C, E-G) => Optimal: Pairings 1, 4 -- $7



If MR Solution 2 (A-B) => Optimal: Pairings 2, 3 -- $6



Approach

- In the sequential approach, given a maintenance routing solution, the crew pairing problem is solved
- We allow the crew scheduler to choose from a collection of maintenance routing solutions
  - Select the one containing the set of short connects that allows the minimum cost crew pairing solution
- Problem: We don't want to solve one crew pairing problem for each maintenance routing solution
- Solution: Extended Crew Pairing Model (ECP)

The Extended Crew Pairing Model (ECP)

- Simultaneously select a cost minimizing set of crew pairings and a corresponding feasible maintenance routing solution from a given set of maintenance routing solutions
- Add constraints that allow pairings with a short connect to be selected only if the chosen maintenance solution assigns the same aircraft to that short connect

The Example Again

Flights:

A B C D E F G H

Short Connects:

A-B A-C E-G

- MR solution (x_1) uses short connects A-C and E-G
- MR solution (x_2) uses short connect A-B

- Potential pairings:
  - A-C-D-F (y_1): \$1
  - A-B-E-F (y_2): \$2
  - C-D-G-H (y_3): \$4
  - B-E-G-H (y_4): \$6
- Crew pairing solutions:
  - x_1 => pairings 1, 4: \$7
  - x_2 => pairings 2, 3: \$6

Matrix Representation for the Example

		x_1	x_2	y_1	y_2	y_3	y_4		rhs	
Flights:		0	0	1	1	0	0	=	1	A
		0	0	0	1	0	1	=	1	B
		0	0	1	0	1	0	=	1	C
		0	0	1	0	1	0	=	1	D
		0	0	0	1	0	1	=	1	E
		0	0	1	1	0	0	=	1	F
		0	0	0	0	1	1	=	1	G
		0	0	0	0	1	1	=	1	H
Short Connects:		0	1	0	–1	0	0	≥	0	A-B
		1	0	–1	0	0	0	≥	0	A-C
		1	0	0	0	0	–1	≥	0	E-G
Convexity:		1	1	0	0	0	0	=	1	Conv.

Notation

- P^k is the set of feasible pairings for fleet family k
- F^k is the set of flights assigned to fleet family k
- T^k is the set of short connects for the flights assigned to fleet family k
- S^k is the set of feasible MR solutions for the flights assigned to fleet family k

Notation, cont.

- δ_{fp} is 1 if flight f is included in pairing p, else 0
- α_{ts} is 1 if MR solution s includes short connect t, else 0
- β_{tp} is 1 if short connect t is contained in pairing p, else 0
- c_p is the cost of pairing p

Notation, cont.

- x_s is a binary decision variable with value 1 if MR solution s is chosen, else 0
- y_p is a binary decision variable with value 1 if pairing p is chosen, else 0

ECP Formulation

$$\min \sum_{p \in P^k} c_p y_p$$

$$st$$

Flights:

$$\sum_{p \in P^k} \delta_{fp} y_p = 1 \qquad \forall f \in F^k$$

Short Connects:

$$\sum_{s \in S^k} \alpha_{ts} x_s - \sum_{p \in P^k} \beta_{tp} y_p \geq 0 \qquad \forall t \in T^k$$

Convexity:

$$\sum_{s \in S^k} x_s = 1$$

$$x_s, y_p \in \{0,1\} \qquad \forall s, p$$

ECP Enhancements

- By exploiting dominance relationships, can dramatically reduce the **number** of MR columns considered in finding an optimal ECP solution
  - MR1 containing short connects AB, CD, GH *dominates* MR2 containing short connect AB
    - Do not need to include MR2 in ECP
- Theoretical bounds and computational observations
  - Example: 61 flights => >> 25,000 MR solutions => 4 non-dominated MR solutions (bounded by 35)
  - Can find these 4 non-dominated MR solutions by solving 4 MR problems

ECP Enhancements, cont.

- Proof: Can relax the **integrality** of MR columns and still achieve integer solutions:
  - Same number of binary variables as original CP

$$
\begin{array}{ll}
\min \sum_p c_p y_p & \\
st & \\
\sum_{p:f\in p} y_p = 1 & \forall f \\
\sum_{s:t\in s} z_s - \sum_{p:t\in P} y_p \geq 0 & \forall t \\
\sum_s z_s = 1 & \\
\boxed{z_s \in \{0,1\}} & \forall s \\
y_p \in \{0,1\} & \forall p
\end{array}
\equiv
\begin{array}{ll}
\min \sum_p c_p y_p & \\
st & \\
\sum_{p:f\in p} y_p = 1 & \forall f \\
\sum_{s:t\in s} z_s - \sum_{p:t\in P} y_p \geq 0 & \forall t \\
\sum_s z_s = 1 & \\
\boxed{z_s \geq 0} & \forall s \\
y_p \in \{0,1\} & \forall p
\end{array}
$$

LP relaxation of ECP is tighter than LP relaxation of a basic integrated approach

Computational Experiment

- **Problem A**:

Lower bound:	31,396.10
ECP with 16 MR columns:	31,396.10
Optimality gap:	0%

- **Problem B**:

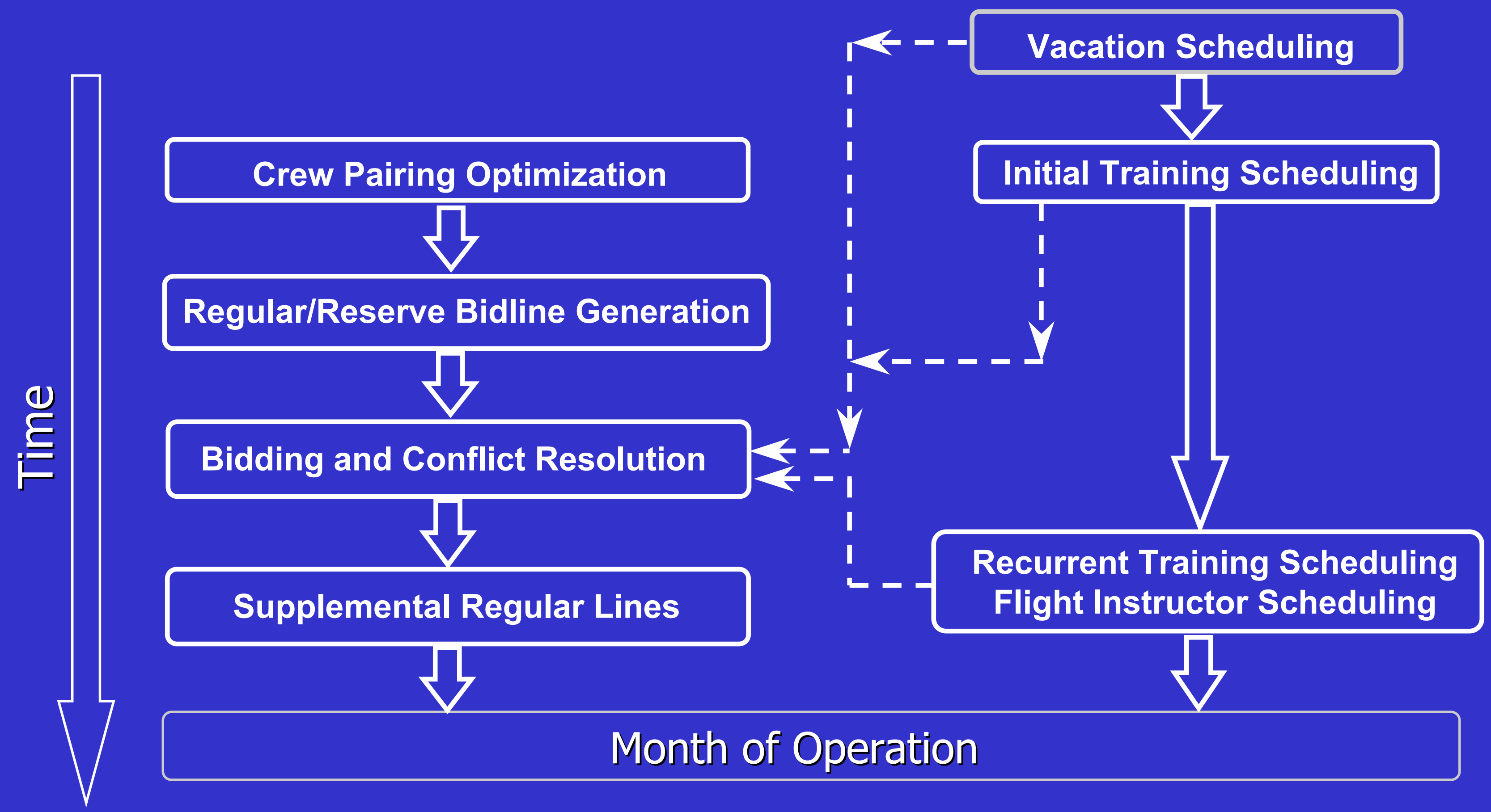
Lower bound:	25,076.60
ECP with 20 MR columns:	25,498.60
Optimality gap:	1.7%

Airline Crew Scheduling Successes

- Excess crew costs in the planning process has been driven to 0-3%
  - AA was 8-10% 15 yrs ago: now 0-2%
  - Each 1% is worth about $10 million/yr
    - 1997 had 9,000 pilots costing $1.2 billion
  - Larger schedules and complex rules

Crew Pre-Month Planning

Ellis Johnson, Georgia Tech



Reserve Demand

Ellis Johnson, Georgia Tech

Net Reserve Demand

Pre-Month Planning

(1) Vacation Conflict
(2) Initial Training Conflict
(3) Transition Conflict
(4) Recurrent Training Conflict

Irregular Operations

(1) Weather Disruptions
(2) Aircraft Maintenance
(3) Sick Leave

Could be up to 25% total trips built

Open Time Trips

Higher Reserve Availability Desired

Voluntary Flying | Premium Flying | Reserves

Could cover 10% of the open trip flying

Crew: The Rest of The Story

Ellis Johnson, Georgia Tech

- Manpower planning, conflicts, over-time flying, and reserves
- In US airlines as high as 30% of the pilots may be on reserve bid lines
  - Actual flying is about 50% of usual
  - Of that flying, more than half is to cover conflicts and as little as 1/3 is to cover disruptions

Conclusions

- Crew scheduling is critical to airline profitability
  – Making maintenance routing decisions independently increases costs
- A model that fully integrates MR and CP can be inflexible and difficult to solve
  – ECP exploits that only some maintenance routing information is relevant and uses dominance to reduce the size of the problem
- More work to be done… especially post-pairing optimization