NETWORK PROBLEMS

1.224J/ESD.204J
TRANSPORTATION OPERATIONS, PLANNING AND CONTROL: CARRIER SYSTEMS

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Announcements

– C. Barnhart open office hours from 1:00-2:30 on Wednesday this week

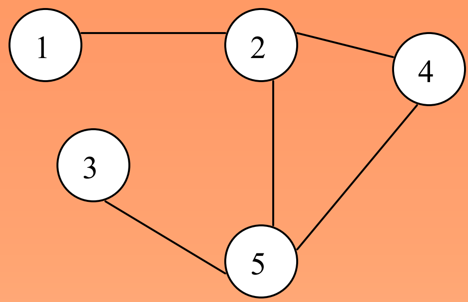
– Reference: Network Flows: Theory, Algorithms, and Applications (Ahuja, Magnanti, Orlin)

Outline

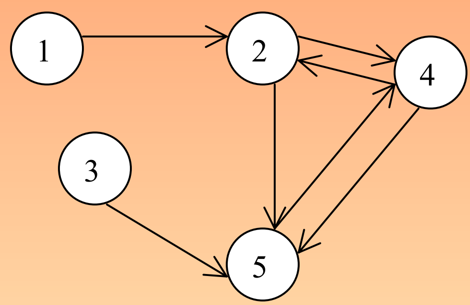
- Network Introduction
- Properties of Network Problems
- Minimum Cost Flow Problem
- Shortest Path
- Maximum Flow Problem
- Assignment Problem
- Transportation Problem
- Circulation Problem
- Multicommodity Flow problem
- Matching Problem

Networks

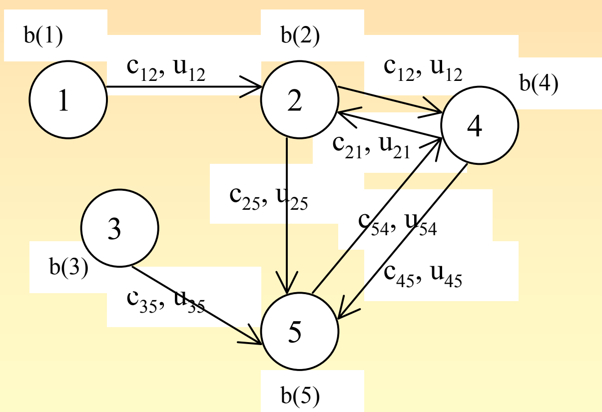
- Very common in transportation
- Can be physical networks: transportation networks such as railroads and highways)
- Network flows sometimes also arise in surprising ways (problems that on the surface might not appear to involve any networks at all).
- Sometimes the nodes and arcs have a temporal dimension that models activities that take place over time. Many scheduling applications have this flavor (crew scheduling; location and layout theory; warehousing and distribution; production planning and control)



Undirected Graph



Directed Graph



Network

Graphs & Networks

- Network problems are defined on graphs
  - Undirected and directed graphs G=(N, A)
    - N=set of nodes
    - A=set of feasible links/ arcs
  - Trees (connected graph; no cycles)
  - Bipartite graphs: two sets of nodes with arcs that join only nodes between the 2 sets
- Additional numerical information such as:
  - *b(i)* representing supply and demand at each node *i*;
  - u_{ij} representing the capacity of each arc *ij*
  - l_{ij} representing the lower bound on flow for each arc *ij*
  - c_{ij} representing the cost of each arc *ij*.

Formulating network flow problems

$$\text{Minimize} \sum_{(i,j)\in A} C_{ij} X_{ij}$$

$s.t$

$$\sum_{\{j:(i,j)\in A\}} X_{ij} - \sum_{\{j:(j,i)\in A\}} X_{ij} = b(i), \forall i \in N \text{............}(1)$$

$$X_{ij} \geq L_{ij}, \forall (i,j) \in A \text{..................................}(2)$$

$$X_{ij} \leq U_{ij}, \forall (i,j) \in A \text{.................................}(3)$$

$$X_{ij} \in Z^{+}, \forall (i,j) \in A \text{.................................}(4)$$

- Concise formulation
- Node arc incidence matrix:
- Columns = arcs
- Rows = nodes
- Outgoing arc $\Rightarrow$ +1
- Incoming arc $\Rightarrow$ -1
- No arc incident $\Rightarrow$ 0
- Sum of rows of A = 0

(1) Balance constraints: flow out minus flow in must equal the supply/demand at the node
(2) Flow lower bound constraints (usually lower bound is not stated and equal to 0)
(3) Capacity constraints
(4) Integrality constraints

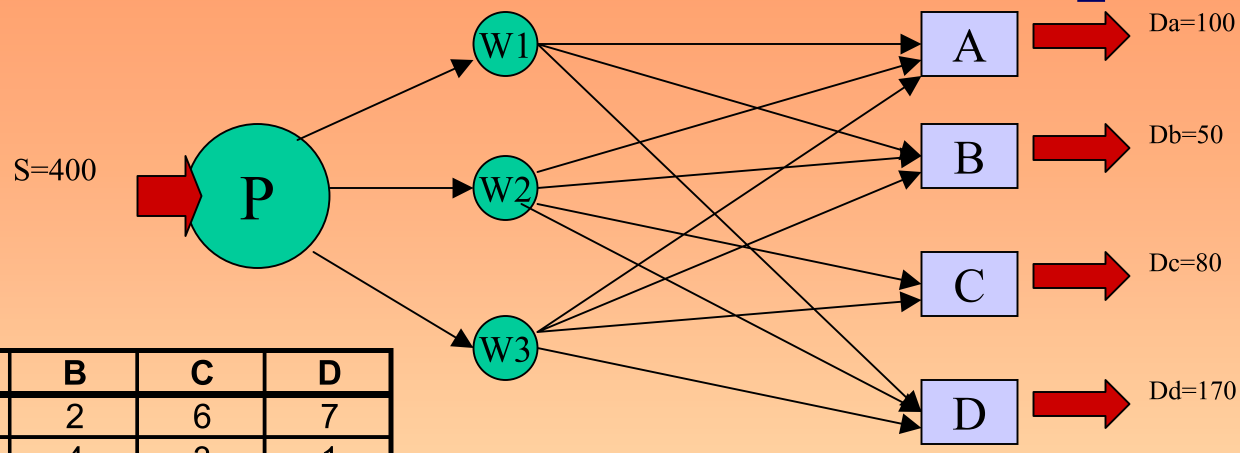
Properties of Network Problems

- Solving the LP relaxation of network problems with integer problem data, yields an integer solution
- Network problems are special cases of LPs and any algorithm for a LP can be directly applied.
- Network flow problems have a special structure which results in substantial simplification of general methods (Network Simplex)
- Many algorithms used to solve network problems (discussed in 1.225)
- Multicommodity problems are more complicated

Minimum Cost Flow Problem

- Objective: determine the least cost movement of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes.
- Applications:
  - distribution of a product from manufacturing plants to warehouses, or from warehouses to retailers;
  - the flow of raw material and intermediate goods through the various machining stations in a production line;
  - the routing of vehicles through an urban street network;
- G=(N,A): directed network defined by a set N of nodes and a set A of m directed arcs.
  - C_{ij}: cost per unit flow on arc $(i,j) \in A$.
  - U_{ij}: capacity; maximum amount that can flow on arc (i,j)
  - L_{ij}: lower bound; minimum amount that must flow on the arc
  - B(i): supply or demand at node $i \in N$
    - If b(i)>0 => node *i* is a supply node
    - If b(i)<0 => node *i* is a demand node
    - If b(i)=0 => node *i* is a transshipment node
  - X_{ij}: decision variables; represent the quantity of flow on arc $(i,j) \in A$

Example: Minimum Cost Flow Warehouse distribution problem



Cij	A	B	C	D
W1	5	2	6	7
W2	4	4	3	1
W3	3	8	5	3

Company A currently serves its 4 customers from 3 warehouses. It costs $\$c_{ij}$ to transport a unit from warehouse *i* to the customer *j*. Transportation from the plant P to the warehouses is free. Transportation of the products from the warehouse to the customers is done by truck. Company A cannot send more than 100 units of product from each warehouse to each customer. Finally, there is a demand for D_j units of the product in region *j*.

Company A would like to determine how many units of product they should store at each warehouse and how many units of product to send from each warehouse to each customer, in order to minimize costs.

Approach

- Decision variables?
  - X_{ij}= flow on each arc

- Objective Function: $MIN \sum_{(i,j)\in A} C_{ij} * x_{ij}$

- Conservation of Flow Constraints:

$$\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b_i, \forall i \in N \qquad x_{ij} \geq 0$$

- Arc capacity constraints:

There is a limit such that a given warehouse-customer route (i,j) can be used by at most u_{ij} units.

$$x_{ij} \leq u_{ij}, \forall (i,j) \in A$$

Warehouse Distribution-Formulation

		Node- Arc Matrix															
	Cij	0	0	0	5	2	6	7	4	4	3	1	3	8	5	3	
		P-W1	P-W2	P-W3	W1-A	W1-B	W1-C	W1-D	W2-A	W2-B	W2-C	W2-D	W3-A	W3-B	W3-C	W3-D	Di
Nodes	P	-1	-1	-1													-400
	W1	1			-1	-1	-1	-1									0
	W2		1						-1	-1	-1	-1					0
	W3			1									-1	-1	-1	-1	0
	A				1				1				1				100
	B					1				1				1			50
	C						1				1				1		80
	D							1				1				1	170

$$Min(5X_{1A} + 2X_{1B} + 6X_{1C} + 7X_{1D} + 4X_{2A} + 4X_{2B} + 3X_{2C} + 1X_{2D} + 3X_{3A} + 8X_{3B} + 5X_{3C} + 3X_{3D}$$

$s.t.$

$$-X_{P1} - X_{P2} - X_{P3} = -400$$

$$X_{P1} - X_{1A} - X_{1B} - X_{1C} - X_{1D} = 0$$

$$X_{P2} - X_{2A} - X_{2B} - X_{2C} - X_{2D} = 0$$

$$X_{P3} - X_{3A} - X_{3B} - X_{3C} - X_{3D} = 0$$

$$X_{1A} + X_{2A} + X_{3A} = 100$$

$$X_{1B} + X_{2B} + X_{3B} = 50$$

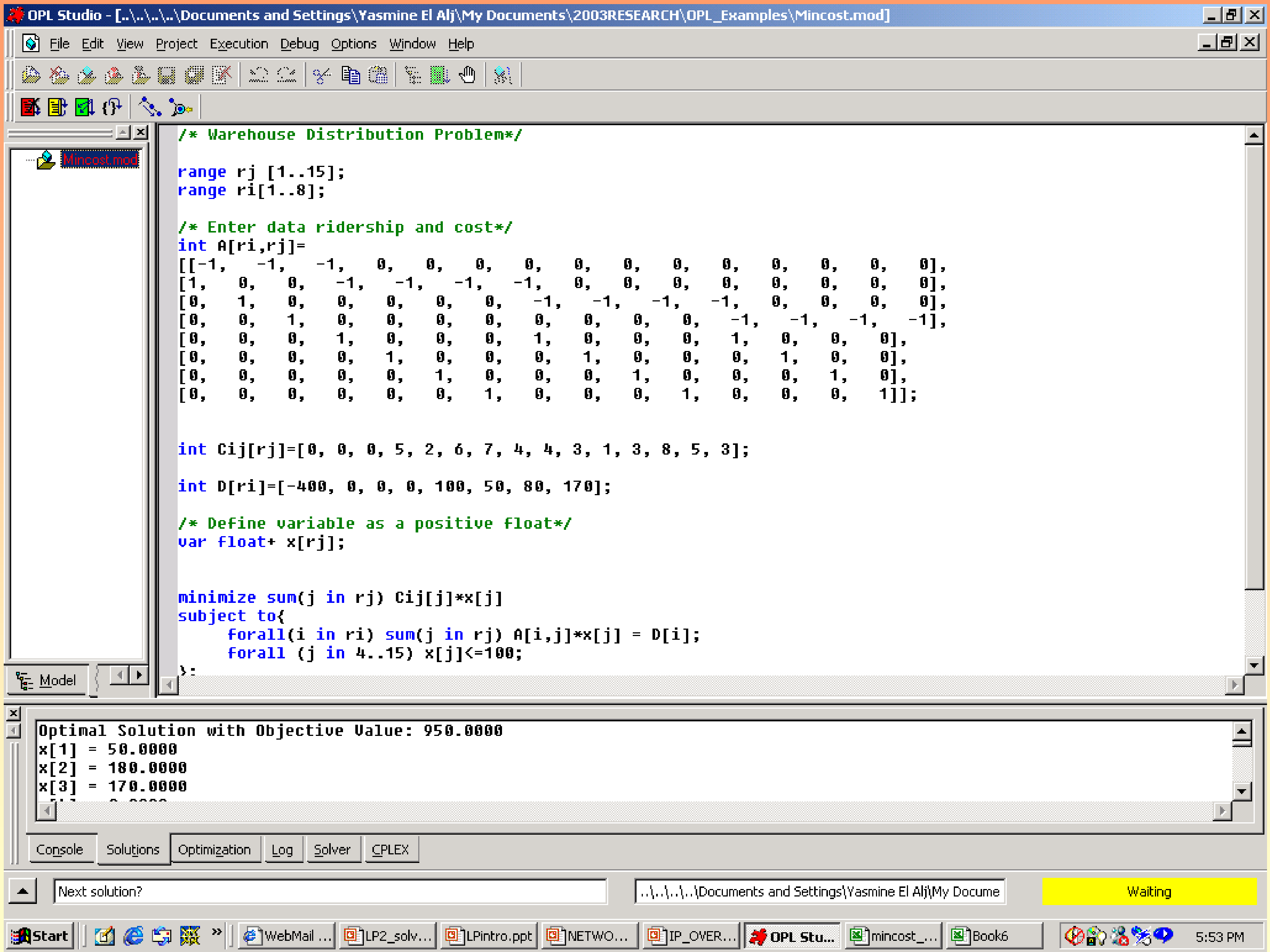
$$X_{1C} + X_{2C} + X_{3C} = 80$$

$$X_{1D} + X_{2D} + X_{3D} = 170$$

$$X_{1A}, X_{1B}, X_{1C}, X_{1D,} X_{2A}, X_{2B}, X_{2C}, X_{2D,} X_{3A}, X_{3B}, X_{3C}, X_{3D} \leq 100$$

$$X_{P1}, X_{P2}, X_{P3}, X_{1A}, X_{1B}, X_{1C}, X_{1D,} X_{2A}, X_{2B}, X_{2C}, X_{2D,} X_{3A}, X_{3B}, X_{3C}, X_{3D} \geq 0$$

In OPL Studio…



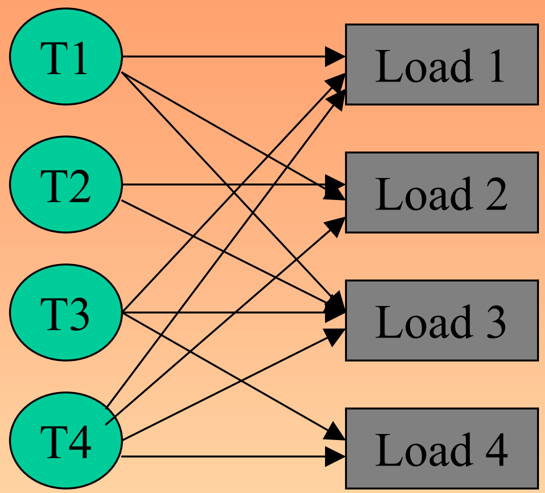
Transportation Problem

- Objective: Transport the goods from the suppliers to the consumers at minimum cost given that:
  - there are m suppliers and n consumers (m can be different from n).
  - The i^{th} supplier can provide s_i units of a certain good and the j^{th} consumer has a demand for d_j units.
  - We assume that total supply equals total demand.
- Applications: distribution of goods from warehouses to customers.
- $G=(N1 \cup N2, A)$: directed network defined by a set N1+N2 of nodes and a set A of $m*n$ directed arcs.
  - N1: supply nodes; N2: demand nodes; $|N1|=m$; $|N2|=n$;
  - $(i,j)\in A$ such that $i\in N1$ and $j\in N2$
  - C_{ij}: unit transportation cost of transporting goods from supplier i to consumer j, per unit flow on arc $(i,j) \in A$.
  - U_{ij}: capacity ; maximum amount that can flow on arc (i,j)
  - L_{ij}: lower bound; minimum amount that must flow on the arc
  - B(i):
    - $b(i)= s_i$ for all $i \in N1$
    - $b(i)= -d_j$ for all $j \in N2$
  - X_{ij}: decision variables; represent the quantity of goods flowing from i to j

Assignment Problem

- Objective: Pair, at minimum possible cost, each object in set N1 with exactly one object in set N2.
- Special case of the transportation problem where number of suppliers equals number of customers and each supplier has unit supply, each consumer has unit demand.
- Applications: assigning people to projects, truckloads to truckers, jobs to machines, tenants to apartments, swimmers to events in a swimming set, school graduates to internships.
- G=(N1 U N2,A): directed network defined by a set N1+N2 of nodes and a set A of *m* directed arcs.
  - $|N1|=|N2|=m$;
  - $(i,j) \in A$ such that $i \in N1$ and $j \in N2$
  - C_{ij}: cost per unit flow on arc $(i,j) \in A$.
  - $U_{ij}:=1$ for all $(i,j) \in A$
  - L_{ij}: lower bound; minimum amount that must flow on the arc
  - $B(i)$: supply or demand at node $i \in N$
    - $B(i) = 1$ for all $i \in N1$
    - $B(i) = -1$ for all $i \in N2$
  - X_{ij}: decision variables; represent the quantity flow on arc $(i,j) \in A$

Assignment Problem



Cij	1	2	3	4
1	6	4	5	n/a
2	n/a	3	6	n/a
3	5	n/a	4	3
4	7	5	5	5

Trucking company TC needs to pick up 4 loads of products at different locations. Currently, 4 truck drivers are available to pick up those shipments. The cost of having driver *i* pick-up shipment *j* is illustrated in the above table.

Formulate the problem of assigning each driver to a load, in order to minimize costs

Assignment problem-Formulation

$$Min(6X_{11}+4X_{12}+5X_{13}+3X_{22}+6X_{23}+5X_{31}+4X_{33}+3X_{34}+7X_{41}+5X_{42}+5X_{43}+5X_{44})$$

$s.t.$

$$-X_{11}-X_{12}-X_{13}=-1$$

$$-X_{22}-X_{23}=-1$$

$$-X_{31}-X_{33}-X_{34}=-1$$

$$-X_{41}-X_{42}-X_{43}-X_{44}=-1$$

$$X_{11}+X_{31}+X_{41}=1$$

$$X_{12}+X_{22}+X_{42}=1$$

$$X_{13}+X_{23}+X_{33}+X_{43}=1$$

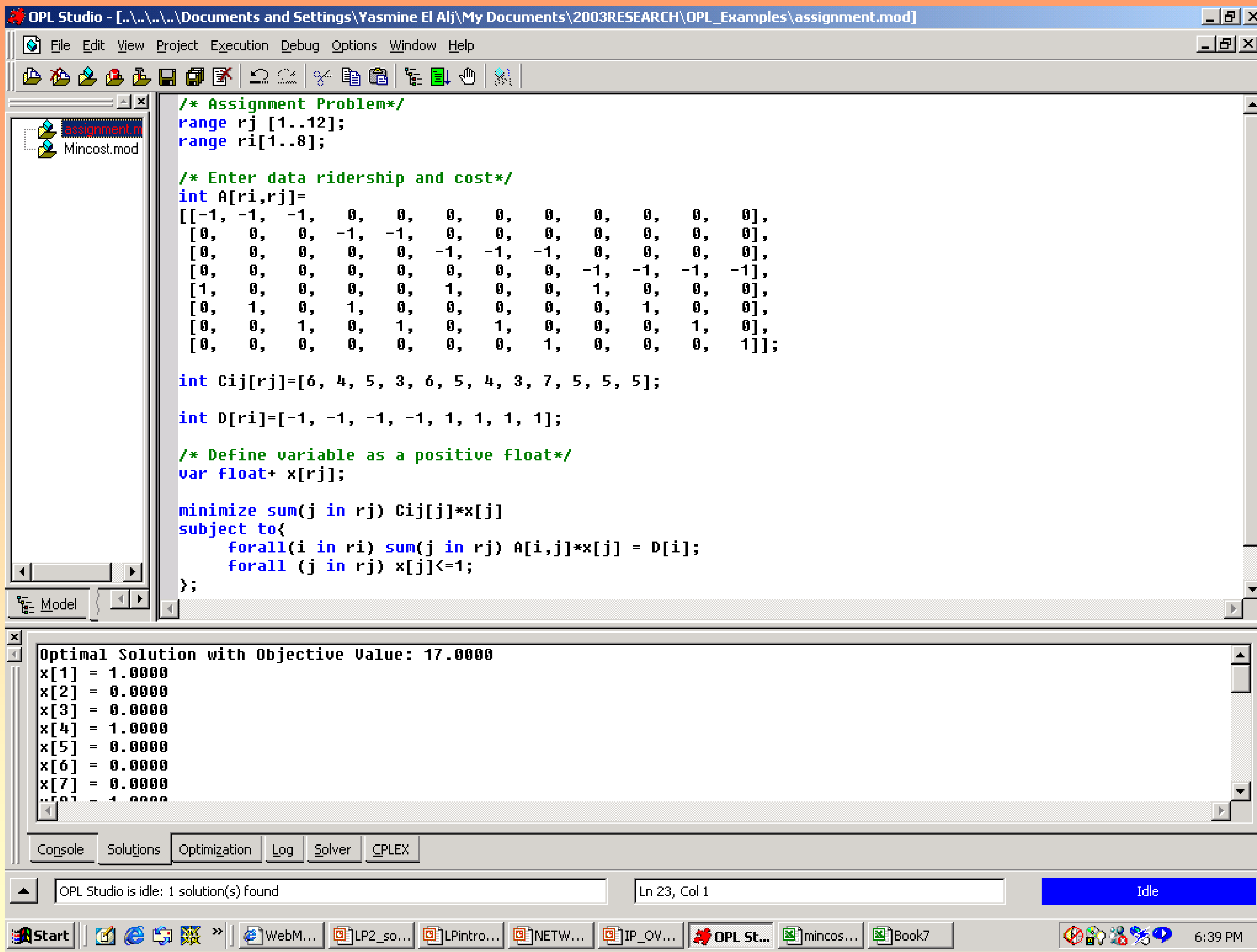
$$X_{34}+X_{44}=1$$

$$X_{11},X_{12},X_{13},X_{22,}X_{23},X_{31},X_{33},X_{34,}X_{41},X_{42},X_{43},X_{44}\le 1$$

$$X_{11},X_{12},X_{13},X_{22,}X_{23},X_{31},X_{33},X_{34,}X_{41},X_{42},X_{43},X_{44}\ge 0$$

		Node- Arc Matrix												
	Cij	6	4	5	3	6	5	4	3	7	5	5	5	
		1-1	1-2	1-3	2-2	2-3	3-1	3-3	3-4	4-1	4-2	4-3	4-4	bi
Nodes	T1	-1	-1	-1										-1
	T2				-1	-1								-1
	T3						-1	-1	-1					-1
	T4									-1	-1	-1	-1	-1
	L1	1					1			1				1
	L2		1		1						1			1
	L3			1		1		1				1		1
	L4								1				1	1

In OPL Studio



Shortest Path

•Objective: Find the path of minimum cost (or length) from a specified source node *s* to another specified sink *t*, assuming that each arc $(i,j) \in A$ has an associated cost (or length) c_{ij}.

• Applications:

• project scheduling; cash flow management; message routing in communication systems; traffic flow through congested city.

• G=(N,A): directed network defined by a set N of nodes and a set A of m directed arcs.

–C_{ij}: cost per unit flow on arc $(i,j) \in A$.

–U_{ij}: capacity ; maximum amount that can flow on arc (i,j)

–L_{ij}:=0 for all (i,j)

–b(i): supply or demand at node $i \in N$

•b(s)=1 ; b(t)=-1;

• b(i)=0 for all other nodes

–X_{ij}: decision variables; represent the quantity flow on arc $(i,j) \in A$

• If want to determine shortest path from source to all every other node in the system, then: b(s)=(n-1); b(i)=-1 for all other nodes.

Example: Product Distribution

A producer wishes to distribute a sample of its product to product testers. In order to minimize the cost of this distribution, it has decided to use its existing supply network. The goal then is to minimize the distance traveled by each sample.

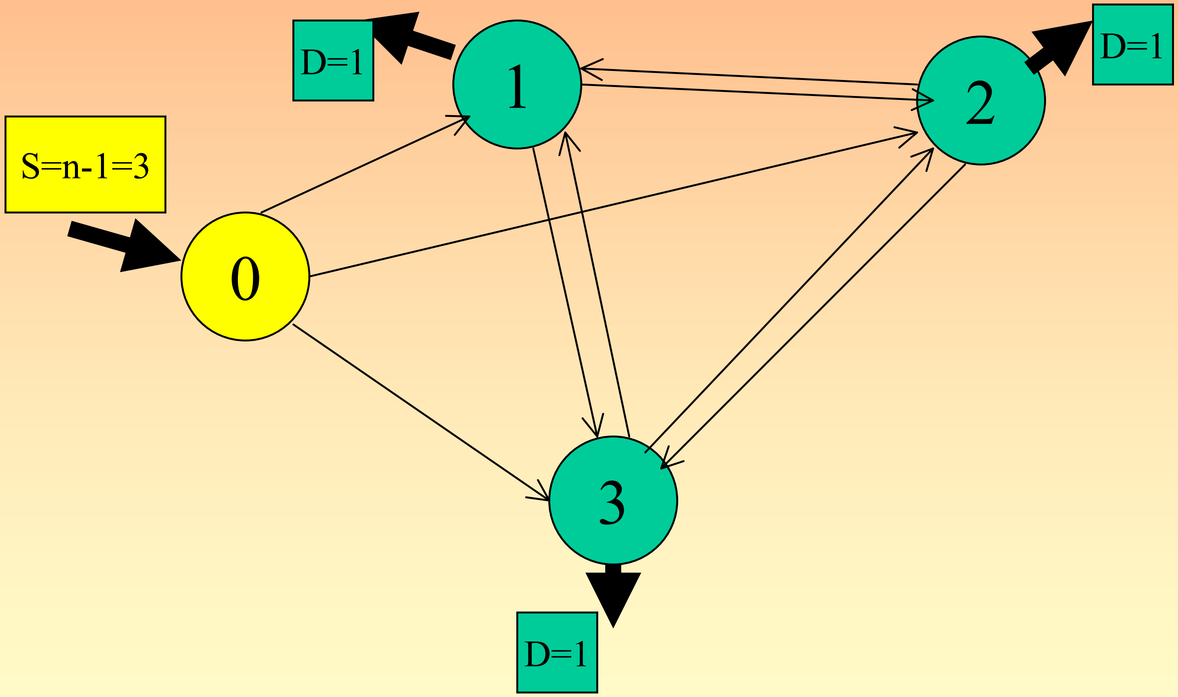
The network G consists of a set of nodes, N, between which the distribution is non-stop. Let |N|=n; we want to deliver a sample to each of the *n-1* cities in our distribution network. We denote the set of these connections, or arcs, as A, and the travel time along such a connection, starting at some point i and connecting to a point j, by T_{ij}.

If we number the plant as node 0, provide a formulation of the problem given that we must deliver only one sample to each destination.

Example: Network Representation

Network Representation n=4

Equivalent to shortest path problem



Product Distribution: Formulation

- Decision variables?
  - X_{ij}= flow on each arc

- Objective Function: $MIN \sum_{(i,j)\in A} T_{ij} * x_{ij}$

- Constraints: (conservation of flow and flow non-negativity)

$$\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b_i, \forall i \in N \qquad x_{ij} \geq 0$$

Product Distribution--Application

$N=\{0,1,2,3\}$

	COSTS	TO j		
	Tij	1	2	3
FROM i	0	16	24	55
	1	n/a	32	30
	2	15	n/a	17
	3	13	11	n/a

	Node-Arc Matrix									
COSTS Tij	16	24	55	32	30	15	17	13	11	
Node/Arcs	0-1	0-2	0-3	1-2	1-3	2-1	2-3	3-1	3-2	bi
0	-1	-1	-1	0	0	0	0	0	0	-3
1	1	0	0	-1	-1	1	0	1	0	1
2	0	1	0	1	0	-1	-1	0	1	1
3	0	0	1	0	1	0	1	-1	-1	1

$$MIN(16x_{01}+24x_{02}+55x_{03}+32x_{12}+30x_{13}+15x_{21}+17x_{23}+13x_{31}+11x_{32})$$

$$s.t.$$

$$-1x_{01}-1x_{02}-1x_{03}+0x_{12}+0x_{13}+0x_{21}+0x_{23}+0x_{31}+0x_{32}=-3$$

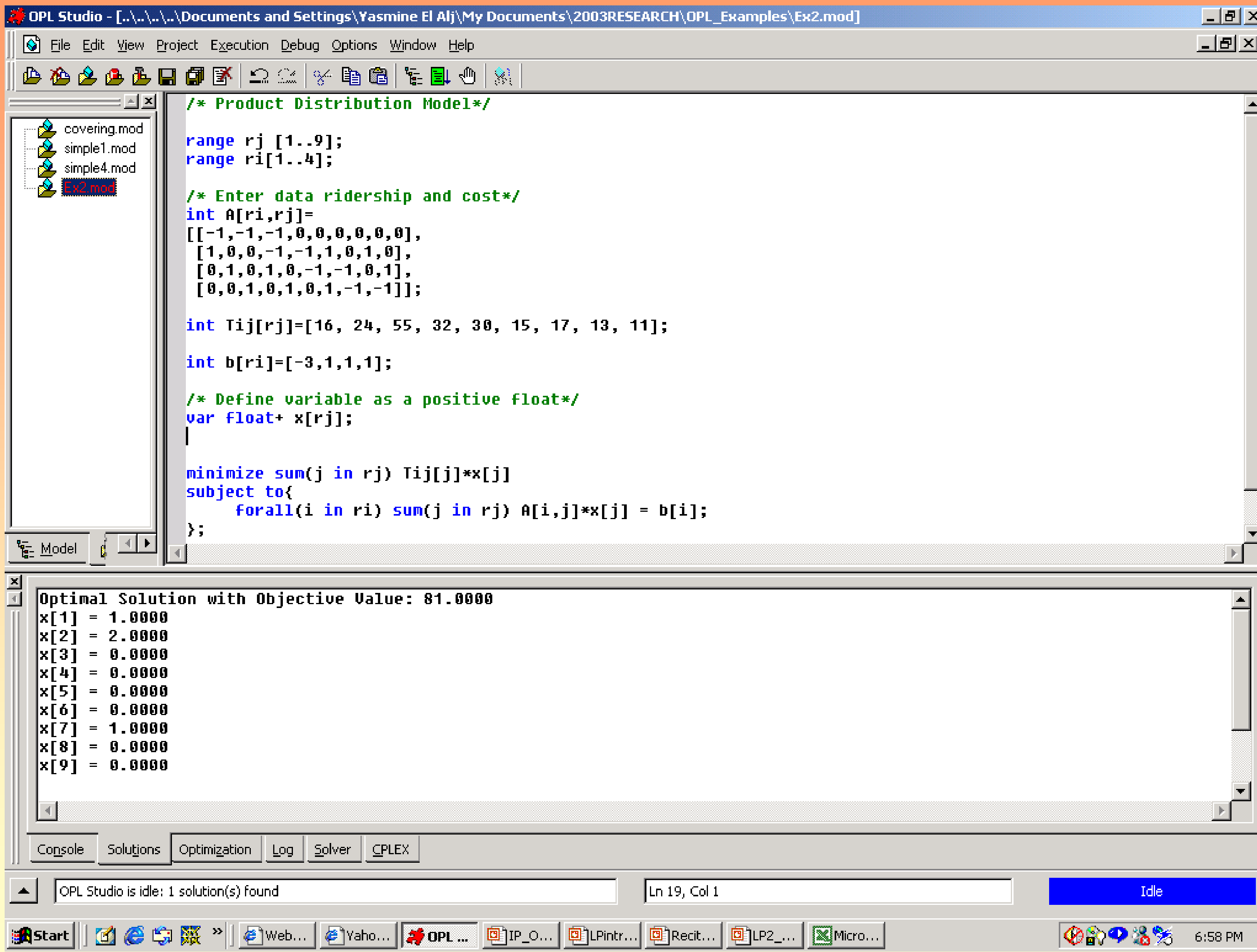
$$1x_{01}+0x_{02}+0x_{03}-1x_{12}-1x_{13}+1x_{21}+0x_{23}+1x_{31}+0x_{32}=1$$

$$0x_{01}+1x_{02}+0x_{03}+1x_{12}+0x_{13}-1x_{21}-1x_{23}+0x_{31}+1x_{32}=1$$

$$0x_{01}+0x_{02}+1x_{03}+0x_{12}+1x_{13}+0x_{21}+1x_{23}-1x_{31}-1x_{32}=1$$

$$x_{01},x_{02},x_{03},x_{12},x_{13},x_{21},x_{23},x_{31},x_{32}\geq 0$$

In OPL…


OPL Studio - [..\..\..\..\Documents and Settings\Yasmine El Alj\My Documents\2003RESEARCH\OPL_Examples\Ex2.mod]
File Edit View Project Execution Debug Options Window Help
covering.mod
simple1.mod
simple4.mod
Ex2.mod
Model
/* Product Distribution Model*/
range rj [1..9];
range ri[1..4];
/* Enter data ridership and cost*/
int A[ri,rj]=
[[-1,-1,-1,0,0,0,0,0,0],
[1,0,0,-1,-1,1,0,1,0],
[0,1,0,1,0,-1,-1,0,1],
[0,0,1,0,1,0,1,-1,-1]];
int Tij[rj]=[16, 24, 55, 32, 30, 15, 17, 13, 11];
int b[ri]=[-3,1,1,1];
/* Define variable as a positive float*/
var float+ x[rj];
minimize sum(j in rj) Tij[j]*x[j]
subject to{
forall(i in ri) sum(j in rj) A[i,j]*x[j] = b[i];
};
Optimal Solution with Objective Value: 81.0000
x[1] = 1.0000
x[2] = 2.0000
x[3] = 0.0000
x[4] = 0.0000
x[5] = 0.0000
x[6] = 0.0000
x[7] = 1.0000
x[8] = 0.0000
x[9] = 0.0000
Console Solutions Optimization Log Solver CPLEX
OPL Studio is idle: 1 solution(s) found
Ln 19, Col 1
Idle
Start
Web…
Yaho…
OPL …
IP_O…
LPintr…
Recit…
LP2_…
Micro…
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Maximum Flow Problem

- Objective: Find a feasible flow that sends the maximum amount of flow from a specified source node *s* to another specified sink *t*.
- Maximum flow problem incurs no cost but is restricted by arc/ node capacities.
- Applications:
  - determining the maximum steady state flow of petroleum products in a pipeline network, cars in a road network, messages in a telecom network; electricity in an electrical network.

- G=(N,A): directed network defined by a set N of nodes and a set A of m directed arcs.
  - $C_{ij}=0$ for all (i,j) in A; introduce arc from *t* to *s* such that $C_{ts}=-1$
  - U_{ij}; U_{ts}= unlimited
  - L_{ij}: lower bound
  - $b(i)=0$ for all $i \in N$
  - X_{ij}: decision variables; represent the quantity of flow on arc $(i,j) \in A$
- Solution: maximize the flow on arc (t,s), but any flow on arc (t,s) must travel from node *s* to node *t* through the arcs in A [because each b(i)=0]. Thus, the solution to the minimum cost flow problem will maximize the flow from *s* to *t*.

Circulation Problem

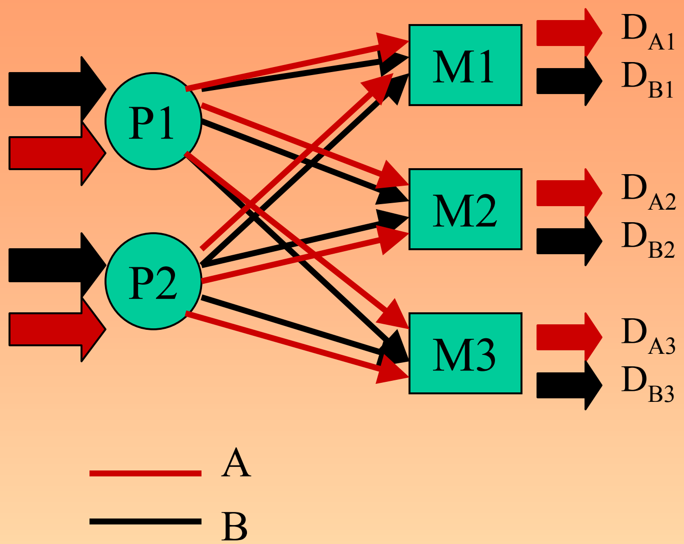
- Objective: Find a feasible flow that honors the lower and upper bounds L_{ij} and U_{ij} imposed on arc flows X_{ij}. Since we never introduce any exogenous flow into the network or extract any flow from it, all the flow circulates around the network. We wish to find the circulation that has the minimum cost.
- Minimum cost flow problem with only transshipment nodes.
- Applications: design of routing schedule for a commercial airline where bound $L_{ij}=1$ on arc (i,j) if airline needs to provide service between cities *i* and *j* , and so must dispatch an airplane on this arc (actually the nodes will represent a combination of both a physical location and a time of day).
- G=(N,A): directed network defined by a set N of nodes and a set A of m directed arcs.
  - C_{ij}: cost per unit flow on arc $(i,j)\in A$.
  - U_{ij}: capacity ; maximum amount that can flow on arc (i,j)
  - L_{ij}: lower bound; minimum amount that must flow on the arc
  - $B(i) = 0$ for all $i\in N$
  - X_{ij}: decision variables; represent the quantity flow on arc $(i,j)\in A$

Multicommodity Flow Problem

- Minimum cost flow problem => deals with single commodity over a network.
- Multicommodity flow problems => several commodities use the same underlying network.
  - Objective: Allocate the capacity of each arc to the individual commodities in a way that minimizes overall costs.
  - Commodities may either be differentiated by their physical characteristics or simply by their origin destination pairs.
  - Different commodities have different origins and destinations, and commodities have separate balance constraints at each node.
  - Commodities share the arc capacities
- Applications: transportation of passengers from different origins to different destinations within a city; routing of non-homogeneous tankers; worldwide shipment of different varieties of grains from countries that produce grains to countries that consume it; the transmission of messages in a communication network between different origin-destination pairs.

➢ MCF problems are network flow problems with side constraints (integrality property doesn't hold)

Multicommodity Flow



	Cij	M1	M2	M3	Supply
Product A	**P1**	6	4	5	150
	P2	5	3	6	130
Product B	**P1**	10	3	9	140
	P2	8	4	6	170
	Demand A	100	70	110	
	Demand B	150	30	130	

A company produces 2 types of products A and B at 3 plants (P1, P2, P3). It then ships these products to 3 market zones (M1, M2, M3). For k=1,2; i=1,2 and j=1,..,3 the following data is given:

- The costs of shipping one unit of product k from plant i to zone j

- The maximum number of units that can be shipped from each plant to each market zone

- The demand for product k at market zone j

- The distribution channel P1-M1 and P2-M3 cannot carry more than 10 and 35 units respectively

Formulate the problem of minimizing transportation costs as an LP

Multi-commodity Flow

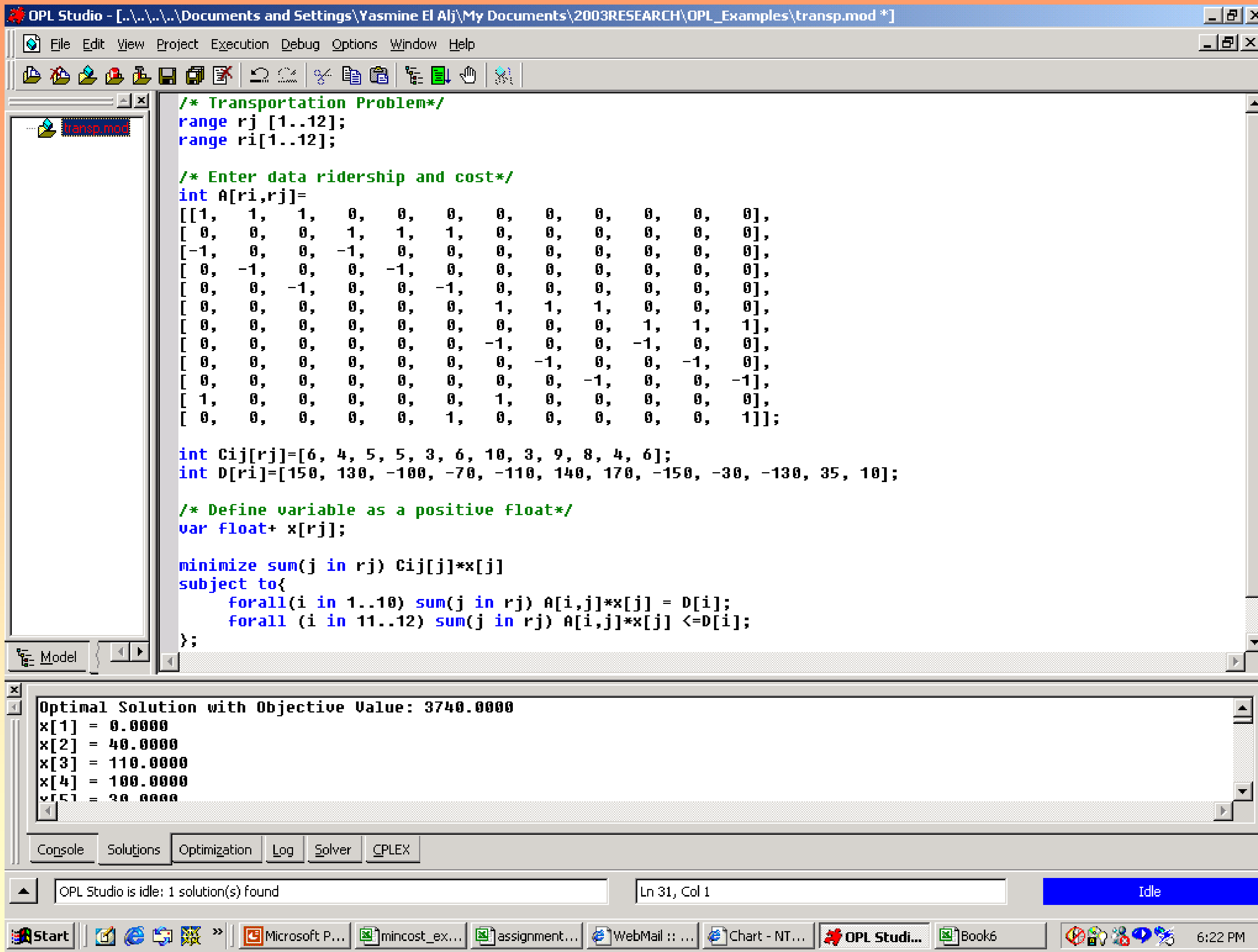
$$Min(6X_{A,P1M1} + 4X_{A,P1M2} + 5X_{A,P1M3} + 5X_{A,P2M1}$$
$$+ 3X_{A,P2M2} + 6X_{A,P2M3} + 10X_{B,P1M1} + 3X_{B,P1M2}$$
$$+ 9X_{B,P1M3} + 8X_{B,P2M1} + 4X_{B,P2M2} + 6X_{B,P2M3})$$
$$s.t.$$
$$X_{A,P1M1} + X_{A,P1M2} + X_{A,P1M3} = 150$$
$$X_{B,P1M1} + X_{B,P1M2} + X_{B,P1M3} = 140$$
$$X_{A,P2M1} + X_{A,P2M2} + X_{A,P2M3} = 130$$
$$X_{B,P2M1} + X_{B,P2M2} + X_{B,P2M3} = 170$$
$$X_{A,P1M1} + X_{A,P2M1} + X_{A,P3M1} = 100$$
$$X_{B,P1M1} + X_{B,P2M1} + X_{B,P3M1} = 150$$
$$X_{A,P1M2} + X_{A,P2M2} + X_{A,P3M2} = 70$$
$$X_{B,P1M2} + X_{B,P2M2} + X_{B,P3M2} = 30$$
$$X_{A,P1M3} + X_{A,P2M3} + X_{A,P3M3} = 110$$
$$X_{B,P1M3} + X_{B,P2M3} + X_{B,P3M3} = 130$$
$$X_{A,P1M1} + X_{B,P1M1} \le 35$$
$$X_{A,P2M3} + X_{B,P2M3} \le 10$$
$$X_{A,ij}, X_{B,ij} \ge 0$$

$$Min(\sum_k \sum_i \sum_j C_{k,ij} X_{k,ij})$$
$$\sum_j X_{k,ij} = S_{k,i}, \forall k = \{A,B\}, \forall i = \{P1,P2\}$$
$$\sum_i X_{k,ij} = D_{k,j}, \forall k = \{A,B\}, \forall j = \{M1,M2,M3\}$$
$$\sum_k X_{k,P1M1} \le 35$$
$$\sum_k X_{k,P2M3} \le 10$$
$$X_{k,ij} \ge 0, \forall i \in \{P1,P2\}, \forall j \in \{M1,M2,M3\}, \forall k \in \{A,B\}$$

Multi-commodity

		Node- Arc Matrix													
		Product A						**Product B**							
	Cij	6	4	5	5	3	6	10	3	9	8	4	6		
		1-1	**1-2**	**1-3**	**2-1**	**2-2**	**2-3**	**1-1**	**1-2**	**1-3**	**2-1**	**2-2**	**2-3**		**bi**
Product A	**P1**	1	1	1										=	150
	P2				1	1	1							=	130
	M1	-1			-1									=	-100
	M2		-1			-1								=	-70
	M3			-1			-1							=	-110
Product B	**P1**							1	1	1				=	140
	P2										1	1	1	=	170
	M1							-1			-1			=	-150
	M2								-1			-1		=	-30
	M3									-1			-1	=	-130
A&B	**Cap P1-M1**	1						1						<=	35
	Cap P2-M3						1						1	<=	10

In OPL…


OPL Studio - [..\..\..\..\Documents and Settings\Yasmine El Alj\My Documents\2003RESEARCH\OPL_Examples\transp.mod *]
File Edit View Project Execution Debug Options Window Help
transp.mod
/* Transportation Problem*/
range rj [1..12];
range ri[1..12];
/* Enter data ridership and cost*/
int A[ri,rj]=
[[1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0],
[-1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0],
[0, -1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0],
[0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1],
[0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0, 0],
[0, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1, 0],
[0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, -1],
[1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1]];
int Cij[rj]=[6, 4, 5, 5, 3, 6, 10, 3, 9, 8, 4, 6];
int D[ri]=[150, 130, -100, -70, -110, 140, 170, -150, -30, -130, 35, 10];
/* Define variable as a positive float*/
var float+ x[rj];
minimize sum(j in rj) Cij[j]*x[j]
subject to{
forall(i in 1..10) sum(j in rj) A[i,j]*x[j] = D[i];
forall (i in 11..12) sum(j in rj) A[i,j]*x[j] <=D[i];
};
Model
Optimal Solution with Objective Value: 3740.0000
x[1] = 0.0000
x[2] = 40.0000
x[3] = 110.0000
x[4] = 100.0000
Console
Solutions
Optimization
Log
Solver
CPLEX
OPL Studio is idle: 1 solution(s) found
Ln 31, Col 1
Idle
Start
Microsoft P...
mincost_ex...
assignment...
WebMail :: ...
Chart - NT...
OPL Studi...
Book6
6:22 PM

Matching problem

- Objective: The matching seeks a matching that optimizes some criteria.
- A matching in a graph G=(N,A) is a set of arcs with the property that every node is incident to at most one arc in the set; thus a matching induces a pairing of some of the nodes in the graph using the arcs in A. In a matching each node is matched with at most one other node, and some nodes might not be matched with any other node.
- Bipartite matching problems: matching problems on bipartite graphs (graphs with two distinct sets of nodes): assignment and transportation problem

Set Partitioning

Example

A company needs to hire extra drivers to run a special shuttle service on Saturday. The shuttle service will last from 6 AM to 6 PM, with shifts of 3 hours minimum. The transit company has found 3 potential drivers. Drivers want to be compensated as follows:

	Shift Type	Hours	Cost ($/hr)	Total Cost
Driver A	1	6-12	20	120
	2	12-18	19	114
Driver B	3	6-9	22	66
	4	9-12	17	51
	5	12-15	18	54
	6	15-18	19	57
	7	6-12	19	114
	8	12-18	18	108
Driver C	9	6-9	19	57
	10	9-12	22	66
	11	12-18	18	108
	12	6-12	20	60
	13	9-15	21	126
	14	15-18	19	57

What should the company do in order to minimize its costs ?

Model

Let S be the set of four 3-hour shifts (6-9; 9-12; 12-15; 15-18).

Let P be the set of 14 combinations (a combination is defined as the combination of driver and rate charged).

Let X_j=1 if package *j* is chosen, 0 otherwise

Let δ_{ij}=1 if shift *i* is covered by package *j*, 0 otherwise.

The only constraint is that every shift should be worked by somebody.

$$Min \quad \sum_{j=1}^{14} C_j X_j$$

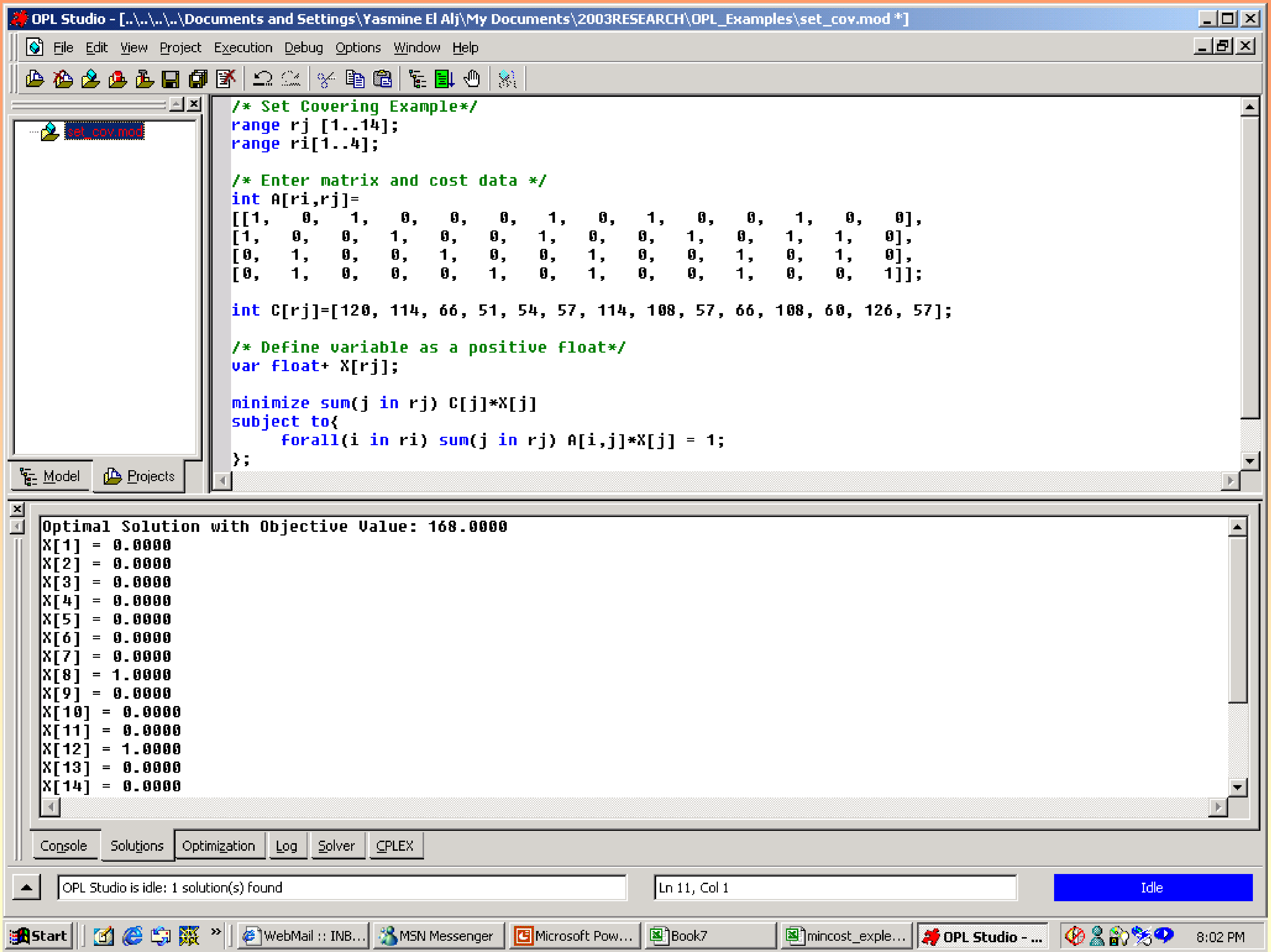
$$s.t.$$

$$\sum_{j=1}^{14} \delta_{ij} X_j = 1, \forall i \in S$$

$$X_j \in \{0,1\}, \forall j \in P$$

		Node- Arc Matrix															
		A		B						C							
	Cost	120	114	66	51	54	57	114	108	57	66	108	60	126	57		
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**		**bi**
Shifts	**6-9**	1		1				1		1			1			=	1
	9-12	1			1			1			1		1	1		=	1
	12-15		1			1			1			1		1		=	1
	15-18		1				1		1			1			1	=	1

In OPL…


OPL Studio - [..\..\..\..\Documents and Settings\Yasmine El Alj\My Documents\2003RESEARCH\OPL_Examples\set_cov.mod *]
File Edit View Project Execution Debug Options Window Help
set_cov.mod
/* Set Covering Example*/
range rj [1..14];
range ri[1..4];
/* Enter matrix and cost data */
int A[ri,rj]=
[[1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0],
[1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0],
[0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0],
[0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1]];
int C[rj]=[120, 114, 66, 51, 54, 57, 114, 108, 57, 66, 108, 60, 126, 57];
/* Define variable as a positive float*/
var float+ X[rj];
minimize sum(j in rj) C[j]*X[j]
subject to{
forall(i in ri) sum(j in rj) A[i,j]*X[j] = 1;
};
Model
Projects
Optimal Solution with Objective Value: 168.0000
X[1] = 0.0000
X[2] = 0.0000
X[3] = 0.0000
X[4] = 0.0000
X[5] = 0.0000
X[6] = 0.0000
X[7] = 0.0000
X[8] = 1.0000
X[9] = 0.0000
X[10] = 0.0000
X[11] = 0.0000
X[12] = 1.0000
X[13] = 0.0000
X[14] = 0.0000
Console
Solutions
Optimization
Log
Solver
CPLEX
OPL Studio is idle: 1 solution(s) found
Ln 11, Col 1
Idle
Start
WebMail :: INB...
MSN Messenger
Microsoft Pow...
Book7
mincost_exple...
OPL Studio - ...
8:02 PM