1.685J/2.034J/18.377J Nonlinear Dynamics and Waves Spring 2007

Problem Set No. 3

Out: Thursday, March 15, 2007

Due: Thursday, April 5, 2007 *in class*

Problem 1

Consider the Van der Pol oscillator

$$\ddot{x} + \epsilon(x^2 - 1)\dot{x} + x = 0.$$

As discussed in class, this nonlinear vibratory system has a stable limit cycle which for $0 < \epsilon \ll 1$ may be approximated as

$$x = 2\cos t + O(\epsilon).$$

Your job is to calculate the frequency ω of this limit cycle correct to $O(\epsilon^2)$; i.e., to find ω_2 in the expansion

$$\omega = 1 + \omega_2 \epsilon^2 + \cdots.$$

Problem 2

A nonlinear mass-spring vibratory system in the presence of dry friction and subject to harmonic excitation is governed by the following dimensionless equation

$$\ddot{x} + 2\zeta \ \mathrm{sgn}\ \dot{x} + x + \epsilon x^2 = F \ \cos \frac{\Omega}{\omega_n}\ t$$

where $\frac{\Omega}{\omega_n} \approx 1$ and $F = O(1)$. Assume light damping ($\zeta \ll 1$) and weak nonlinearity ($\epsilon \ll 1$).

Find the appropriate scaling of the small parameter ζ in terms of ϵ so that light damping and weak nonlinear effects are equally important near resonance. What is the width and height of the resonance peak in terms of ϵ?

Note: the spring nonlinearity is quadratic.

Problem 3

Consider a Van der Pol oscillator with a cubic nonlinear spring under harmonic forcing:

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x + \epsilon x^3 = F \cos \frac{\Omega}{\omega_n} t.$$

Assuming $0 < \epsilon \ll 1$, analyze the response for

(a) $F = O(\epsilon)$ and $\frac{\Omega}{\omega_n} \approx 1$

(b) $F = O(1)$ and $\frac{\Omega}{\omega_n} \approx 3$

(c) $F = O(1)$ and $\frac{\Omega}{\omega_n} \approx \frac{1}{3}$.