Problem Set No. 5

Out: Thursday, April 26, 2007

Due: Thursday, May 10, 2007 *in class*

Consider an infinite uniform beam of cross-sectional area A, mass per unit length ρ, and flexural rigidity EI. The beam rests on a nonlinear elastic foundation which exerts a restoring force per unit length

$$\alpha_1 \psi + \alpha_3 \psi^3 \quad (\alpha_1 > 0),$$

where ψ is the beam deflection at x, t, and α_1, α_3 are constants.

(a) Assuming that, apart from the nonlinear behavior of the foundation, the rest of the system behaves linearly, derive the governing equation of motion for $\psi(x, t)$.

(b) Write the governing equation in dimensionless form.

(c) Derive the nonlinear dispersion relation, for small-amplitude periodic waves.

(d) Derive the nonlinear Schrödinger equation for this system and investigate the stability of the periodic waves found in (c) to large-scale modulations.