

Application Example 4 (Bayes' Theorem)

EARTHQUAKE PREDICTION FROM IMPERFECT PREMONITORY SIGNS

Ordinary people as well as seismologists have observed that, in some cases, major earthquakes occur shortly after certain anomalous events, which they then have claimed can be used for earthquake prediction. One of the earliest reported such premonitory signs is the anomalous behavior of animals. Recently, interest has shifted towards more objectively measurable phenomena such as geophysical anomalies, variations in groundwater level, and small changes in the topography near the causative earthquake fault. Mathematical models have also been developed, trying to establish theoretical links between such quantitative observables and the occurrence of large earthquakes.

The main issue that determines the practical usefulness of these premonitory events is the accuracy with which predictions can be made. Accuracy can be quantified in terms of the following probabilities

$$\begin{aligned} P_{1|1} &= P[\text{earthquake occurs}|\text{earthquake is predicted}] \\ P_{0|1} &= P[\text{earthquake does not occur}|\text{earthquake is predicted}] = 1 - P_{1|1} \\ P_{0|0} &= P[\text{earthquake does not occur}|\text{earthquake is not predicted}] \\ P_{1|0} &= P[\text{earthquake occurs}|\text{earthquake is not predicted}] = 1 - P_{0|0} \end{aligned} \tag{1}$$

For a perfect prediction system, $P_{1|1} = P_{0|0} = 1$ and $P_{0|1} = P_{1|0} = 0$.

The probabilities in Eq. 1 depend on the strength of the association between the premonitory event and the occurrence of earthquakes. Unfortunately, this association is often weak. To make a quantitative analysis, define the following events:

$$\begin{aligned}
 E &= \text{earthquake occurs in a given day} \\
 E^C &= \text{earthquake does not occur in a given day} \\
 A, B, C, \dots &= \text{premonitory event of type a, b, c, \dots occurs in} \\
 &\hspace{15em} \text{a given day} \\
 A^C, B^C, C^C, \dots &= \text{premonitory event of type a, b, c, \dots does not} \\
 &\hspace{15em} \text{occur in a given day}
 \end{aligned}
 \tag{2}$$

A typical daily probability for a major earthquake might be $P[E] = 10^{-5}$ (hence $P[E^C] = 1 - 10^{-5}$), meaning that at a given location large earthquakes might occur on average every about 300 years. Also, a typical association between a premonitory event A and large earthquakes might be

$$\begin{aligned}
 P[A|E] &= 0.1 \text{ (meaning that event A occurs in only 10\% of the days} \\
 &\hspace{10em} \text{that precede major earthquakes)} \\
 P[A|E^C] &= 0.001 \text{ (meaning that A occurs on average once every} \\
 &\hspace{10em} \text{1000 of the days that are not followed by an earthquake)}
 \end{aligned}
 \tag{3}$$

Notice that, for prediction purposes, one looks at the probability of A in the 24 hours that precede a major earthquake. Applying Bayes' theorem to this case gives

$$\begin{aligned}
 P_{1|1} &= P[E|A] = P[E] P[A|E]/P[A] \\
 &= 10^{-5} (0.1)/[(0.1)(10^{-5})+(0.001)(1-10^{-5})] \\
 &= 10^{-3}
 \end{aligned}
 \tag{4}$$

Notice that the probability of a major earthquake, which for a generic day is 10^{-5} , increases 100-fold, to 10^{-3} , after observation of the premonitory event A. This increase should be considered significant in a scientific sense, but may not be sufficient to issue warnings of an impending earthquake. After all, only once every 1000 such warnings, an earthquake would actually occur. Also consider that issuing false earthquake warnings is

very costly and that, following two or three such erroneous calls, people would lose confidence in the predictions.

Advocates of earthquake warning have observed that, although single premonitory signs are seldom useful as a basis for issuing warnings, the use of several such signs in combination may lead to more accurate predictions. Suppose for example that two diagnostic events, A and B, are being monitored. For simplicity, assume that, when taken in isolation, A and B have the same association with E, i.e.

$$\begin{aligned} P[A|E] &= P[B|E] = 0.1 \\ P[A|E^C] &= P[B|E^C] = 0.001 \end{aligned} \quad (5)$$

and that, given E or E^C , A and B are independent. Such conditional independence implies

$$\begin{aligned} P[A \cap B|E] &= P[A|E] P[B|E] = (0.1)(0.1) = 0.01 \\ P[A \cap B|E^C] &= P[A|E^C] P[B|E^C] = (0.001)(0.001) = 10^{-6} \end{aligned} \quad (6)$$

Suppose now that an earthquake warning is issued when both A and B occur simultaneously. The probability that the warning is followed by an earthquake is now

$$\begin{aligned} P_{1|1} &= P[E|A \cap B] = P[E] P[A \cap B|E] / P[A \cap B] \\ &= 10^{-5} (0.01) / [(0.01)(10^{-5}) + (10^{-6})(1 - 10^{-5})] \\ &= 0.09 \end{aligned} \quad (7)$$

Hence, the combined use of two independent premonitory events increases the likelihood of issuing correct warnings by a factor of 100, from 0.001 to 0.1. However, Eq. 6 gives $P[A \cap B|E] = 0.01$, meaning that 99% of the earthquakes occur without the combined premonitory sign ($A \cap B$)!

Problem 4.1

(a) What do you conclude from the previous example?

(b) Repeat the calculations in Eqs. 6 and 7 for the case of 1, 2, 3, 4 premonitory events, under the following conditions:

1. all events A, B, C, D have the same probabilities $P[\text{event occurs}|E]$ and $P[\text{event occurs}|E^C]$

2. all events are conditionally independent, given E or E^C

3. repeat the analysis for the following alternative sets of probabilities:

(i) $P[\text{event occurs}|E] = 0.1, P[\text{event occurs}|E^C] = 0.001$ (same as in the previous examples)

(ii) $P[\text{event occurs}|E] = 0.1, P[\text{event occurs}|E^C] = 0.00001$

(iii) $P[\text{event occurs}|E] = 1, P[\text{event occurs}|E^C] = 0.01$

Comment on the results.