Brief Notes #3
Random Variables: Continuous Distributions

- **Continuous distributions**

  - Cumulative distribution function (CDF)

$$F_X(x) = P[X \le x]$$
$$P[x_1 < X \le x_2] = F_X(x_2) - F_X(x_1)$$

  - Average probability density in an interval $[x_1, x_2]$

$$\frac{P[x_1 < X \le x_2]}{x_2 - x_1} = \frac{F_X(x_2) - F_X(x_1)}{x_2 - x_1}$$

  - Probability density function (PDF)

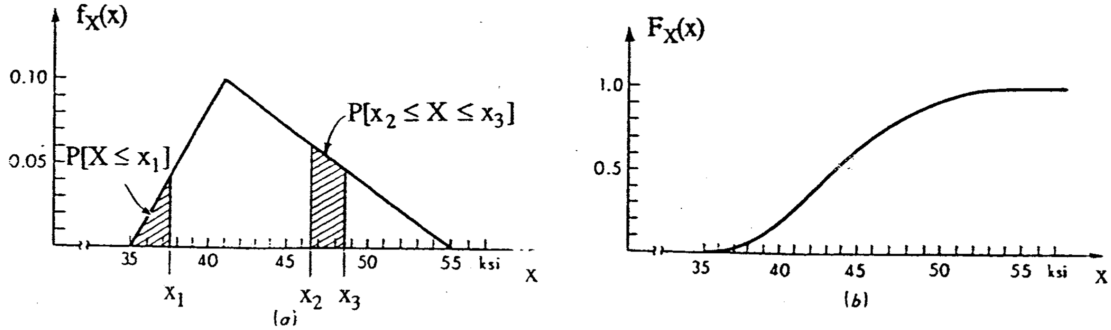
$$f_X(x_1) = \lim_{x_2 \to x_1} \frac{P[x_1 < X \le x_2]}{x_2 - x_1} = \left.\frac{dF_X}{dx}\right|_{x_1^-}$$
$$\int_{x_1}^{x_2} f_X(x)dx = P[x_1 < X \le x_2] = F_X(x_2) - F_X(x_1)$$

  - Properties of the PDF

1. $f_X(x) \ge 0$

2. $\int_{-\infty}^{\infty} f_X(x)dx = 1$

3. $\int_{-\infty}^{u} f_X(x)dx = F_X(u)$



Example of PDF and corresponding CDF of a continuous random variable: steel-yield-stress.
(a) Probability density function; (b) cumulative distribution function.

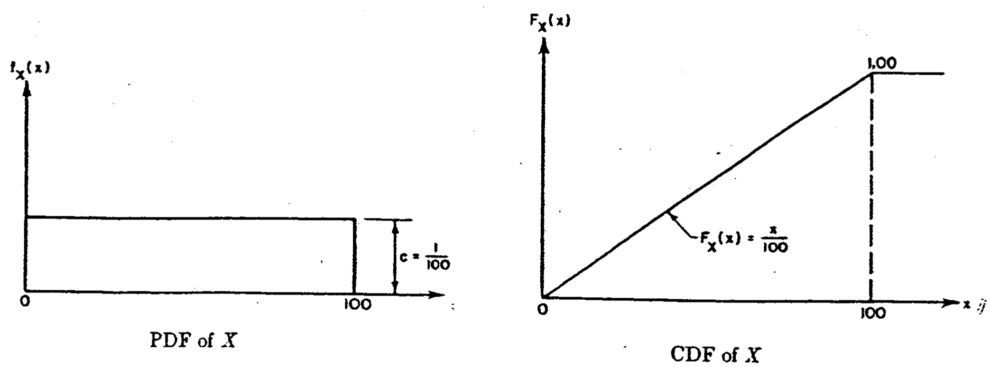
- **<u>Examples of continuous probability distributions</u>**

  - <u>Uniform distribution</u>

$f_X(x) = c$, where c= constant, $a < x < b$
$= 0$, otherwise.

Then, $c = \dfrac{1}{b-a}$

$$F_X(x) = \begin{Bmatrix} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{Bmatrix}$$



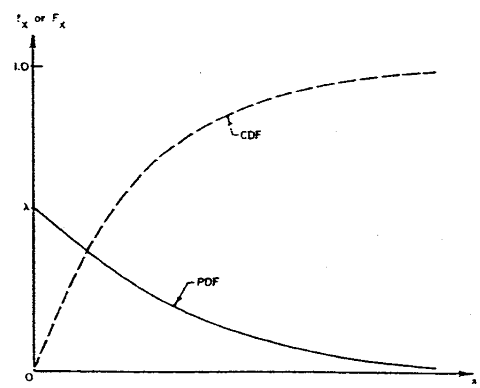
Example of uniform distribution.

• <u>Exponential distribution</u>

Let T = time to first arrival in a Poisson point process

$$F_T(t) = P[T \le t] = 1 - P[T > t]$$
$$= 1 - P[\text{no occurrence in } [0, t]]$$
$$= 1 - e^{-\lambda t}$$

$$f_T(t) = \lambda e^{-\lambda t}, \qquad t \ge 0$$



PDF and CDF of the exponential distribution.