2.25 ADVANCED FLUID MECHANICS Fall 2005

QUIZ 1

THURSDAY, October 13, 2005, 7:00-9:00 P.M.

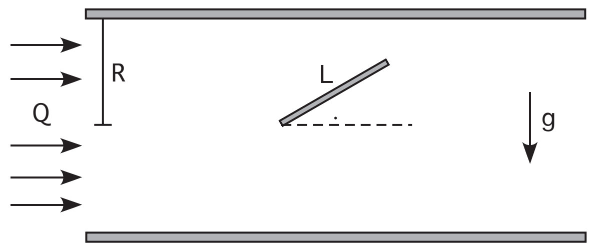
OPEN QUIZ WHEN TOLD AT 7:00 PM

THERE ARE TWO PROBLEMS
OF EQUAL WEIGHT

Please answer each question in SEPARATE books

You may use FOUR (4) pages of handwritten notes as well as the handout on Bernoulli equation plus any tables you feel are necessary

Question 1:



Consider a channel of height $2R$ with rectangular cross-section as shown in the sketch. A hinged plank of a length $L < R$ and at an angle θ is located at the center of the channel. (You may assume that L is small compared to the length perpendicular to the sketch.) The plank has a mass per unit width, m. A constant volume flux per unit width, Q, of an inviscid, incompressible fluid is applied at the inlet of the channel.

a.) Find all equilibrium values for θ (include BOTH stable and unstable equilibria!) To simplify your calculation, you may assume that we have chosen the mass, m, and volume flux, Q, such that for *at least one*equilibrium state, θ is small.

b.) Which values of θ from part (a) represent stable equilibria? Which are unstable? Justify your answers.

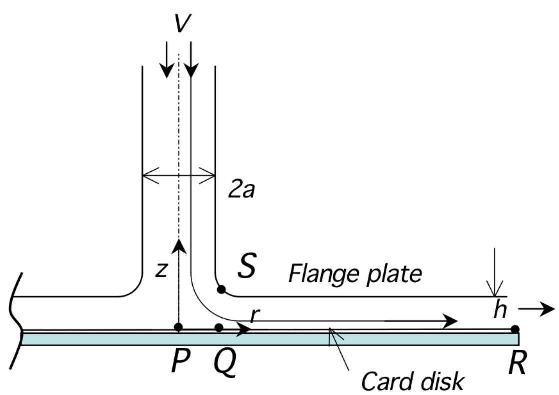
2. Lift Force on a Circular Disk

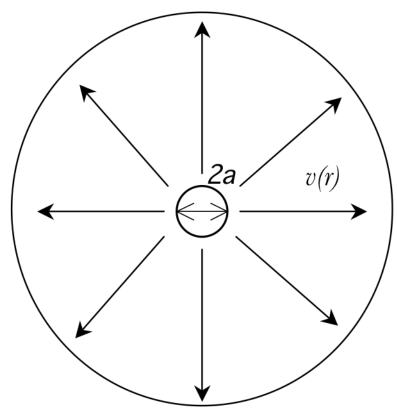
In the movie on "Pressure Fields and Acceleration", you saw a demonstration of the following experiment:

Photo removed for copyright reasons.
See film "Pressure Fields and Fluid Acceleration"
at http://web.mit.edu/fluids/www/Shapiro/ncfmf.html

Compressed air is blown axially down the narrow tube (of radius a) and flows radially outwards below a flange plate (a flat circular disk of radius $R \gg a$). This flange is separated from a circular cardboard disk by a distance $h < a$. As the flow rate is increased, eventually the net 'pressure force' pushing down on the cardboard disk becomes negative and lifts the disk upwards.

(a) The flow in this device is complex, especially in the region where the flow rearranges from axial downflow to radial outflow. However we shall model the streamlines in the flow as shown in the figures below: The average inlet velocity is V and the static pressure is p_0.





The point P is a stagnation point. To find the net pressure force on the disk we need to evaluate the pressure distribution $p(r)$ along the streamline $\overline{PQR}$. Model the flow along $\overline{PQR}$ as inviscid with streamlines that are purely radial with $v_r = f(r)$.

Briefly but clearly answer the following questions:

- Where is the maximum pressure along the streamline $\overline{PQR}$
- Is there any pressure gradient across streamlines in
  - (i) The radial outflow region $(r > a)$?
  - (ii) The complex flow for $r < a$.

- Is the pressure at point S higher or lower than at point Q?

(b) **The Velocity Field**

In the region PQ the air must accelerate radially from zero at P in order to accommodate the additional air arriving from the axial inflow in the tube above. We thus propose that $v_r = C_1 r$ for $r < a$. Show that for any arbitrary radial position $r > a$ the velocity field is of the form

$$v_r = \frac{C_2}{r} \quad \text{for } r > a \tag{2.1}$$

and evaluate the constants C_1 and C_2 in terms of the parameters h, a, V

(c) **Pressure Field**

Using your velocity field, evaluate the gage pressure field $\left(p(r) - p_0\right)$ along the streamline $\overline{PQR}$ and plot your answer graphically, clearly indicating the shapes of the curve you draw and labeling any points of maximum or minimum gage pressure.

Comment briefly on the role of viscosity; if viscous effects are important in this problem, where would their effects be most noticeable?

(d) **Total Force on Disk**

Integrate your answer for $p(r)$ over the entire disk to show that the total <u>downward</u> pressure force acting on the disk is

$$F_p = \frac{1}{2}\pi\rho V^2\left[R^2 - \frac{a^4}{8h^2}\right] + \rho V^2\left(\frac{a^4}{4h^2}\right)\ln\left(a/R\right) \tag{2.2}$$

Explain why this force can be negative, and find the inflow air velocity V required to levitate a 10 gram cardboard disk when $a = 0.2cm$, $R = 5cm$, $h = 0.1cm$.

(e) **Flow following lift-off of the disk**:

Once the force balance described above is achieved, the disk "lifts off" from the lab bench that it is resting on and is attracted to the top flange plate, such that the gap decreases and $h = h(t)$. Use an appropriate form of conservation of mass to find new expressions (for $r \leq a$ and for $r \geq a$) for the unsteady velocity field $v(r,t)$ in the gap. Check that your expressions make sense physically and mathematically.

Derive a nonlinear differential equation for the resulting time-rate of change in the plate separation $\dot{h} \equiv dh/dt$. You do NOT have to integrate this expression! You may approximate the pressure at the exit of the flange plate as remaining at atmospheric pressure throughout; $p(R) = p_0$.