MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

2.26 Spring 2004 — Final Project

Suggested Topics for the Final Course Project
These course projects are designed to allow you to pursue in greater depth a topic related to compressible flows. The course project will consist of a written report and an oral presentation ($\sim$ 15 minutes). You may work in groups or individually. Please let me know by the end of next week (Friday, April 9) the topic of your project and the names of the people in your group. It is important to remember that this is a *technical* paper. It must include some technical analysis related to compressible flow and may include a discussion of physical phenomena, theories, experiments and/or outstanding questions in the field.

Some suggested topics are listed below — you may also choose a topic not listed that runs closer to your own interests.

Review of a "classic" paper You may use an important paper in compressible flow as the "backbone" of your project. The project must include a complete summary of the important points of the paper and a brief review of the important references therein. An outstanding project will also include an analysis that goes beyond the analysis in the paper. This might include a numerical analysis of parameter space not included in the original manuscript, use of theory in a design project, etc. Some good candidates for papers include:

- Coleman and Mizel, "Existence of caloric equations of state in thermodynamics." *J. Chem. Phys.* **40** (1964) 1116–1125.
- G. I. Taylor, "The formation of a blast wave by a very intense explosion." (Parts I, II). *Proc. Royal Soc. A*, **201** (1950).
- Many, many other papers are listed in the bibliography in Thompson.

Whither supersonic flight? Recall all the fastest planes were built 30–40 years ago (an eternity on a technical time scale). Why is this? What technical and economic factors are driving the development of supersonic flight? Remember this must be a *technical* paper. Your analysis must include some technical calculations related to issues involving supersonic flight and/or design.

Granular Flows Understanding processing and transport of granular is becoming increasingly important in many industries including pharmaceuticals and ceramics. Currently our understanding of these systems is limited, and even the fundamental equations of motion are unknown. To devise a continuum model for these systems, it is necessary to include the effects of compressibility as variations in density play an important role in these flows.

Rocket nozzles The design of an effective rocket nozzle requires configuring an expanding high-temperature flow so as to maximize thrust and minimize weight. How do these designs work and what are the important supporting technologies?

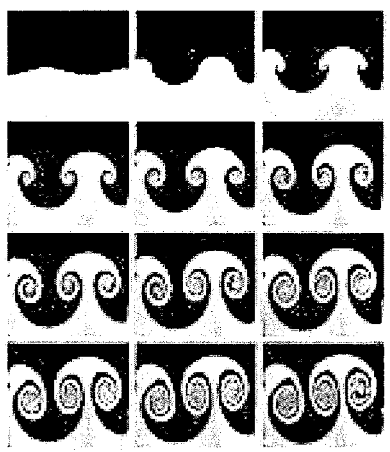
Scramjet (supersonic combustible ramjet) What are they? How do they work? Is this the next big thing in supersonic flight?

Expansion of cooling gases Rank-Hilsch tubes (vortex tubes) and Joule-Thompson refrigerators rely on the tendency of gases to cool when they are expanded. Such devices find significant technical application when modest amounts of cooling are desired without the use of complex mechanical systems. How do these systems work? What are the performance limits of these systems?

Explosions and explosives Explosions are often regarded as strong shock waves. They may arise from combustible gas mixtures, dust-laden gases, solid materials and shaped charges and other reactive media. What are the important technical aspects of explosions and explosives, and how do explosions differ from simple shock waves?

Sonoluminescence and other thermo-acoustic phenomena. Intense sound waves, when focused in a resonating chamber, can lead to substantial localized heating. In sonoluminescence, sound waves are focused on a gas bubble and the energy is converted into light.

Richtmyer-Meshkov instability in shock fronts The contact surface between a strongly accelerated fluid and the one it advances into can be unstable.



`http://www.engr.arizona.edu/newsletters/AESpring02/microgravity.html`

Focused implosions In some situations, such as inertial confinement fusion and high-explosive triggers for atomic bombs, it is desired to create a spherically imploding shock wave. The original studies of such problems date from the Second World War.

Similarity transformations of the compressible boundary layer During much of the 20th century, compressible boundary layers could be analyzed only by analytical techniques. Consequently, significant effort was put into the development of boundary layer similarity transformations that account for variations in density and other physical properties. Most textbooks focus only on incompressible transformations (Blasius or Falkner-Skan). Catalog the various compressible boundary layer transformations (Howarth, Illingworth, Cohen-Reshotko, and many others), and determine how well their assumptions track experimental data.

Image removed due to copyright considerations.

See "Nasa jet smashes speed record," BBC News UK Edition, 28 March 2004.
http://news.bbc.co.uk/1/hi/sci/tech/3575561.stm.

2.26

- News; scram jet! (show movie + O.H.)
- Projects

<u>Detonation Waves</u> (sec. 7.8 and Taylor papers)

G.I. Taylor 1950

Series of photos in Life Magazine. Can we predict r as a function of t?

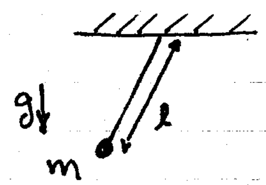
Brief review of dimensional analysis:

<u>Buckingham Pi Thm</u>:

n = # of independent parameters
m = # of "dimensions" (M, L, T)
$k = n - m$ = # dimensionless groups, $\Pi_1, \Pi_2 \ldots$

$$\Rightarrow \Pi_1 = f(\Pi_2, \Pi_3 \ldots)$$

E.g. Pendulum



$$\omega = f(m, g, \ell) \Rightarrow n = 4$$

$$M, L, T \Rightarrow m = 3$$

$$\Rightarrow 1 \text{ dim'less group} : \Pi_1 = \frac{g\omega^2}{\ell}$$

$$\Pi_1 = f(-) = \text{const.}$$

$$\Rightarrow \boxed{\omega \propto \sqrt{\ell/g}}$$

Image removed due to copyright considerations.

"A series of photographs showing the growth of time of the fireball produced
by an atomic-bomb explosion in New Mexico." Originally published in Life Magazine, 1945.

Back to Taylor...

$$r = f(t, \underbrace{\rho}_{\text{density of air}}, \overbrace{\varepsilon}^{\text{energy stored in the bomb}}) \qquad n = 4 \quad m = 3 \quad n-m = 1$$

$$t = [T] \qquad r = [L]$$

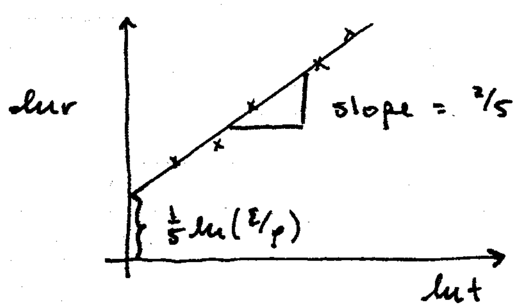
$$\rho = [M/L^3] \qquad \varepsilon = [ML^2/T^2]$$

$$\Pi_1 = \frac{\rho r^5}{\varepsilon t^2} = \text{const.} \qquad \left[\frac{\cancel{M}}{\cancel{L^3}}\,\frac{\cancel{T^2}\,\cancel{L^5}}{\cancel{M}\,\cancel{L^2}\,\cancel{T^2}}\right]$$

$$\Rightarrow \boxed{r \propto \varepsilon^{1/5}\, t^{2/5}\, \rho^{-1/5}}$$

$$\therefore \quad \ln r = \frac{1}{5}\ln\left(\frac{\varepsilon}{\rho}\right) + \frac{2}{5}\ln t$$

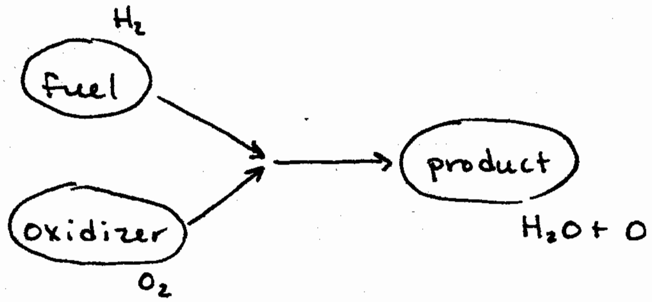
(Taylor shows const ~ 1)

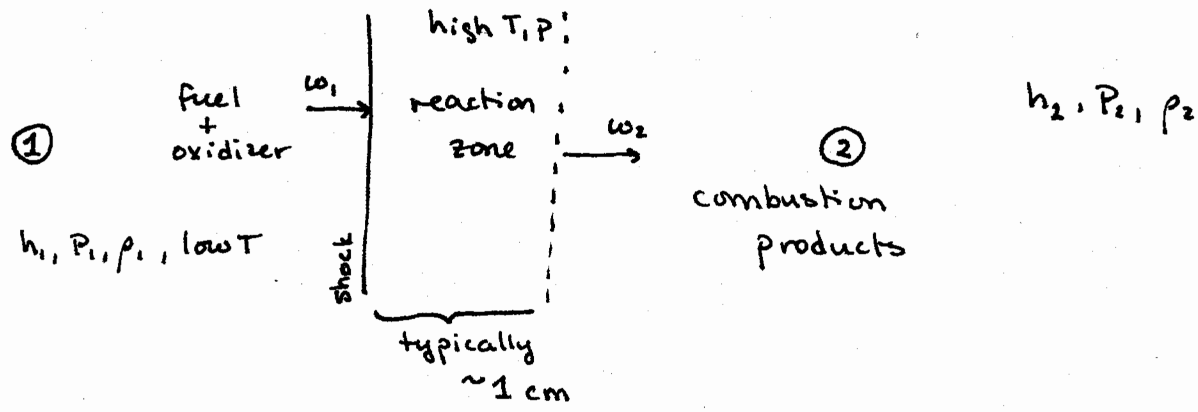


We know ρ for air so from the intercept we can calculate ε!

$$\varepsilon = 8.45 \times 10^{13} \text{ Joules}$$ (energy released in an atomic bomb blast)

"Full" calc. including compressibility + shock calc...



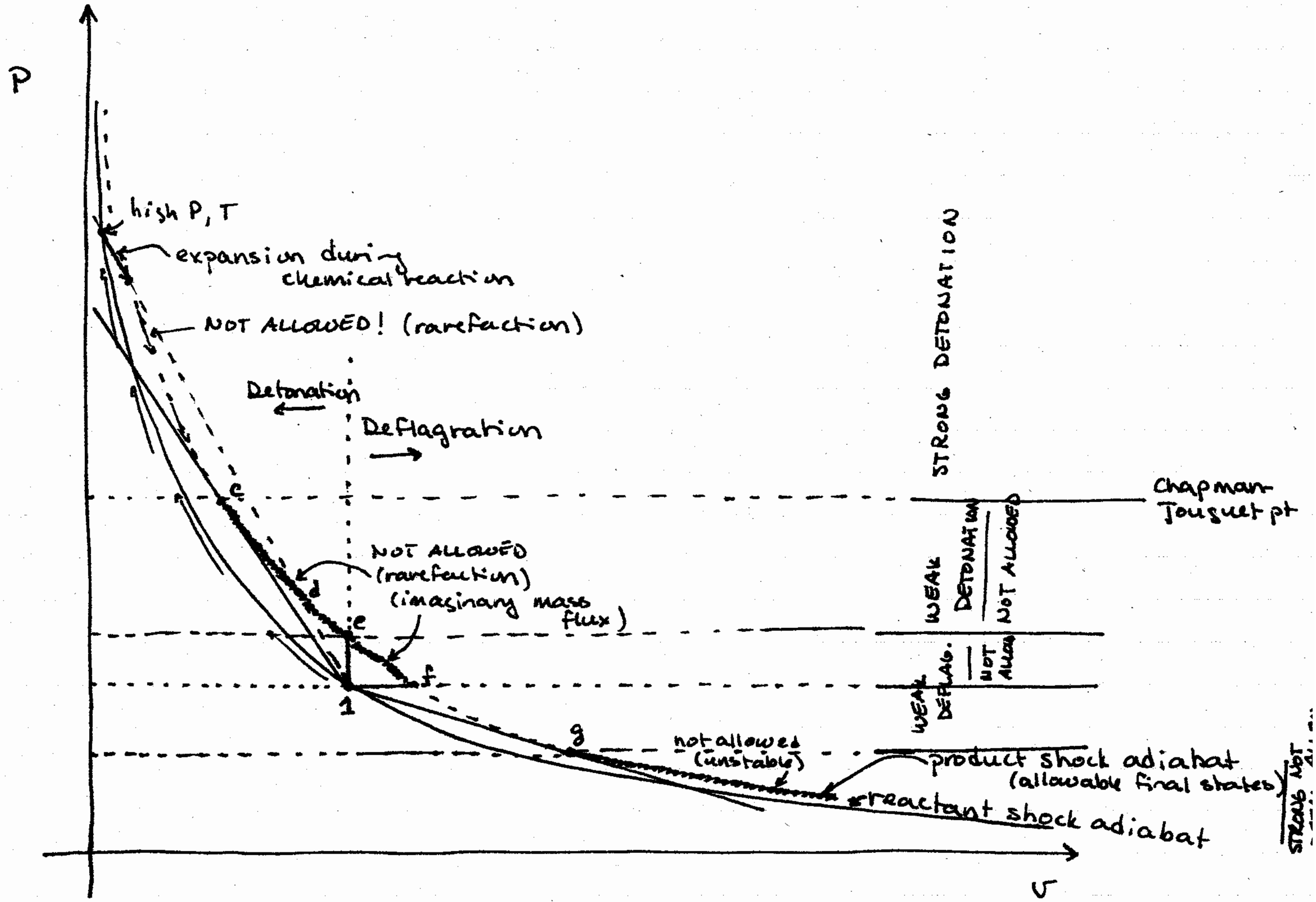


- Mass + momentum still conserved
- Combustion converts chemical energy → kinetic energy

For a perfect gas:
$$\left.\begin{aligned} h_1 &= c_{p_1} T + h_1^\circ \\ h_2 &= c_{p_2} T + h_2^\circ \end{aligned}\right\} \quad h_1^\circ - h_2^\circ = \Delta h^\circ$$
= heat released by combustion

Recall we can rearrange Rankine-Hugoniot + an equation of state to find $P_2 = P_2(v_2)$ (shock adiabat)

and $J^2 = -\dfrac{[P]}{[v]}$ (= slope)



- $h_1(P_1, v_1)$ and $h_2(P_2, v_2)$ are <u>different</u> functions
  ∴ different curves for reactants + products

- c = Chapman-Jouguet pt. (detonation process that occurs most often physically)

- Recall tangent to 1 has slope $\left(\partial P/\partial v\right)_s = -\rho_1^2 c_1^2$
  ↑ isentropic @ 1

- $J^2 = -\dfrac{[P]}{[v]}$

$$-\rho_1^2 c_1^2 > -J^2 = -\rho_1^2 w_1^2 \Rightarrow \boxed{w_1 > c_1}$$

$\therefore$ $\boxed{\text{detonation velocity is always supersonic}}$

Strong Detonation	above c:	$w_1 > c_1$ $\quad w_2 < c_1$
	c (CJ detonation pt.)	$w_1 > c_1$ $\quad w_2 = c_2$
Weak Detonation	c → e:	NOT ALLOWED (rarefaction)
Weak Deflagration	e → f:	NOT ALLOWED (imaginary mass flux)
	f → g:	$w_1 < c_1$ $\quad w_2 < c_2$
	g (CJ deflagration pt.)	$w_1 < c_1$
Strong Deflagration	below g:	NOT ALLOWED (unstable)

Can show that the Chapman-Jouguet pt. is a pt. of <u>minimum entropy</u> for detonation products:

$$\frac{[P]}{[v]} = -J^2 \quad \Rightarrow \quad [v_1 - v_2]\, dJ^2 - J^2 dv_2 = dP_2$$

$$[h + \tfrac{1}{2}w^2] = 0 \quad \Rightarrow \quad dh_2 + J^2 v_2\, dv_2 = \tfrac{1}{2}(v_1^2 - v_2^2)\, dJ^2$$

$$dh_2 = T_2\, ds_2 + v_2\, dP_2$$

$$\underbrace{T_2}_{>0}\, ds_2 = \tfrac{1}{2}\underbrace{(v_1 - v_2)^2}_{>0}\, dJ^2 \quad \Rightarrow \quad \boxed{\frac{ds_2}{dJ^2} > 0}$$

$$\left(\frac{ds_2}{dv_2}\right)_c = \left(\frac{ds_2}{dJ}\right)_c \underbrace{\left(\frac{dJ}{dv_2}\right)_c}_{\substack{\| \\ 0}} = 0 \qquad \therefore \text{ isentrope is tangent to C-J}$$

$$\therefore \quad \underbrace{\left.\frac{\partial P_2}{\partial v_2}\right|_c}_{-J_{min}^2 \,=\, -\rho_2^2 w_2^2} = \left.\frac{\partial P_2}{\partial v_2}\right|_s = -\rho_2^2 c_2^2$$

$\therefore$ $\boxed{w_2 = c_2}$ for C-J pt. (exactly sonic)

$\boxed{w_2 < c_2}$ above C-J pt. (subsonic)

(supersonic below c)

<u>Chapman-Jouguet Detonation</u>

- usually encountered in practice (mimimize entropy production)

<u>In a perfect gas</u>

$[\rho w] = 0$ $\qquad$ $c^2 = \gamma R T$

$[P + \rho w^2] = 0$ $\qquad$ $P = R \rho T$

$[h + \tfrac{1}{2} w^2] = 0$ $\qquad$ $c = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow \rho = \frac{\gamma P}{c^2}$

$$\rho_2 w_2 = \rho_1 w_1$$

$$\frac{\gamma_2 P_2}{c_2^2} w_2 = \frac{\gamma_1 P_1}{c_1^2} w_1$$

(1) $$\boxed{\frac{\gamma_2 P_2}{V_2} M_{2n}^2 = \frac{\gamma_1 P_1}{V_1} M_{1n}^2}$$

$$(2) \qquad P_1 \left(1 + \gamma_1 M_{1n}^2\right) = P_2 \left(1 + \gamma_2 M_{2n}^2\right)$$

$$(3) \qquad \frac{\gamma_2}{\gamma_2 - 1} P_2 v_2 \left(1 + \frac{\gamma_2 - 1}{2} M_{2n}^2\right) = \frac{\gamma_1}{\gamma_1 - 1} P_1 v_1 \left(1 + \frac{\gamma_1 - 1}{2} M_{1n}^2\right) + \Delta h^\circ$$

At the C-J pt. $M_{2n} = 1$; let $\gamma_2 = \gamma_1$

$$M_{1n} = \sqrt{\mathcal{H} + 1} \underbrace{\overbrace{\pm}^{\text{detonation}}}_{\text{deflagration}} \sqrt{\mathcal{H}} \qquad (\text{solve } 1, 2, 3 \text{ for } M_{1n})$$

$$\mathcal{H} \equiv \frac{(\gamma^2 - 1)\Delta h^\circ}{2\gamma R_1 T_1}$$

As $\mathcal{H} \to 0$, $M_{1n} \to 1$ (shock vanishes)

Can also rearrange 1, 2 + 3 to express ratios in terms of M_{1n}.

$$\frac{P_2}{P_1} = \frac{\gamma_1 M_{1n}^2 + 1}{\gamma_2 + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{\gamma_1 (\gamma_2 + 1) M_{1n}^2}{\gamma_2 (1 + \gamma_1 M_{1n}^2)}$$

$$\frac{T_2}{T_1} = \underbrace{\frac{P_2}{P_1}}_{F(M_{1n})} \underbrace{\frac{\rho_1}{\rho_2}}_{G(M_{1n})} \frac{R_2}{R_1}$$

<u>Weak Deflagration</u> (or slow combustion)

Near f, and J^2 is very small,

Flow velocities are small $\Rightarrow$ $P_1 \approx P_2$; $h_1 \approx h_2$

$$C_{p1} T_1 + h^{o1} = C_{p2} T_2 + h^{o2}$$

$$T_2 = \frac{C_{p1}}{C_{p2}} T_1 + \frac{\Delta h^o}{C_p^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{T_1}{T_2} \frac{R_1}{R_2}$$