2.004 Dynamics and Control II
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

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Problem Set 1

Assigned: Feb, 6, 2008 Due: In lab, week of Feb. 11, 2008

Reading:

- Review solution of first-order ODEs (18.03)
- Review elementary rotational dynamics (2.003).
- 2.004 Class Handout: *Description of the Experimental Rotational Plant*

These problems are designed to help you understand the first laboratory experiment.

Problem 1: Consider the basic flywheel rotating in bearings, as shown in Fig. 1 of the handout *Description of the Experimental Rotational Plant*. Assume that the flywheel, spinning with angular velocity $\Omega(t)$, is driven by a time-varying torque source $T(t)$, and that the friction torque $T_f(\Omega)$, from the bearings and the eddy-current damping, is angular-velocity dependent (but not necessarily linear).

Write a differential equation relating the angular velocity Ω to the applied torque $T(t)$.

Problem 2: Modify your differential equation from Problem 1 for the following cases:

(a) The frictional torque is coulomb in nature, that is

$$T_f(\Omega) = T_c \text{sgn}(\Omega)$$

where $\text{sgn}(\Omega)$ is the *signum* function:

$$\begin{aligned} \text{sgn}(\Omega) &= 1 \quad \text{for } \Omega > 0, \\ &= -1 \quad \text{for } \Omega < 0, \end{aligned}$$

and is undefined for $\Omega = 0$, and where T_c is a constant. In other words the torque is constant in magnitude, and only depends on the rotational direction (it acts to oppose the motion). You may assume unidirectional motion.

(b) The frictional torque is viscous, that is

$$T_f(\Omega) = B\Omega$$

(c) The frictional torque has both a coulomb and a viscous component

$$T_f(t) = T_c \text{sgn}(\Omega) + B\Omega$$

Problem 3: Now consider the "spin-down" of the flywheel from an initial condition $\Omega(0) = \Omega_0$ with no applied torque $T(t) = 0$. Solve the three differential equations from Problem 2, and sketch the form of the angular velocity response. On your sketch show what happens as t becomes large.

Problem 4: From the data given in the descriptive handout, compute the mass, weight and moment of inertia (I will be using the symbol J in this course) of the flywheel in SI units. You may ignore the contribution of the inner hub and the spokes.

Problem 5: If you know the value of the moment of inertia J, describe how you might experimentally determine the values of the frictional parameters T_c, and B from a "spin-down" test, in which you spin the flywheel, and record the decay in angular velocity $\Omega(t)$ in the computer, or on a chart.