2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology

Department of Mechanical Engineering

2.004 *Dynamics and Control II*

Spring Term 2008

Problem Set 4

Assigned: Feb. 29, 2008 Due:March 7, 2008

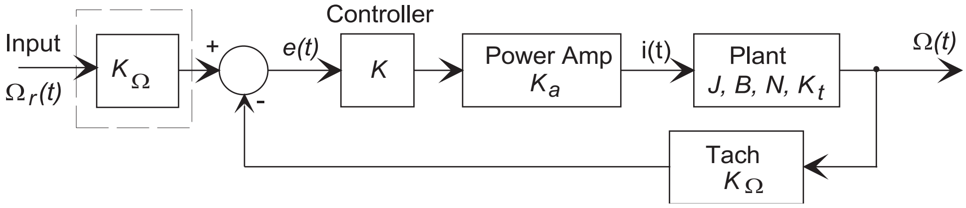
Reading:

- 2.004 Class Handout: *Introduction to the Operational Amplifier*

Problem 1:

Do this problem before your Lab 3 session and take your solution with you.

In Lab 3 you will be designing and implementing a closed-loop controller to regulate the angular velocity of the copper flywheel. In Lab 2 you measured the characteristics of the system components. This problem asks you to predict the closed-loop behavior of the system, so that you may compare your predicted and measured behavior in the lab:



(a) Derive the transfer function relating the angular velocity Ω of the motor to the voltage input command in terms of the controller gain K, servo amp gain K_a, motor torque constant K_t, and tachometer gain K_Ω. Ignore nonlinearities such as Coulomb friction. (Question: Why did I include the block K_Ω, in the input?)

(b) Now put in numerical values:

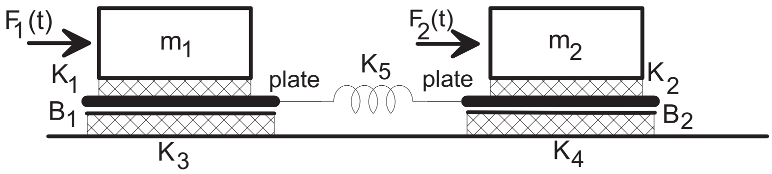
- You computed the value of J.
- The motor-drive gears have 44 and 180 teeth.
- The value of $K_a = 2$ amps/volt. The value of $K_\Omega = 0.016$ v/rpm.
- Assume a value $K = 3$ for the controller.
- We will start with one damping magnet. From Lab. 1, an average over several groups for the damping with one magnet was $B = 0.014$ N.m.s/rad.

(b) With all of the numerical values substituted, except for the controller gain K, derive the closed loop time-constant in terms of K.

(c) Similarly, determine the steady-state error to a constant (step) input as a function of the controller gain K.

Hint: You may find certain similarities between this problem and the automobile cruise control example we discussed in class.

Problem 2: A parts assembly station on a production line exhibits a severe vibration problem. A simplified schematic representation is shown below:



Two large tables of mass m_1 and m_2 are each mounted to a sliding metal plate on resilient rubber mounts, with shear stiffness K_1 and K_2 as shown. The tables are each subjected to a vibrational excitation force, $F_1(t)$ and $F_2(t)$. The plates are able to slide viscously on a second pair of deformable rubber mounts, with shear stiffness K_3 and K_4 as shown. The viscous sliding coefficients are B_1 and B_2. The two plates are coupled by a shaft with longitudinal stiffness K_5.

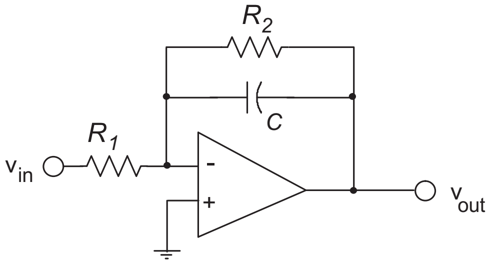
(a) Draw a linear graph for the system using the two forces F_1 and F_2 as inputs.

(b) Write continuity (force balance) equations for each of the nodes on your graph. (You don't have to consider the reference node.)

(c) Using the elemental impedances/admittances substitute the component values and generate a set of simultaneous algebraic equations in the nodal velocities.

Do not attempt to solve the equations.

Problem 3: The op-amp circuit shown below is commonly used as a "filter" in control systems.



In the class handout we showed that the output of the standard inverting amplifier is

$$v_{out} = -\frac{R_f}{R_{in}} v_{in}$$

(a) Extend the argument in the handout to show that for any impedance elements Z_f and Z_{in} the output may be written

$$V_{out}(s) = -\frac{Z_f}{Z_{in}} V_{in}(s).$$

(b) Find the transfer function $V_{out}(s)/V_{in}(s)$ for the circuit above.

(c) Find the differential equation relating $v_{out}(t)$ to $v_{in}(t)$.

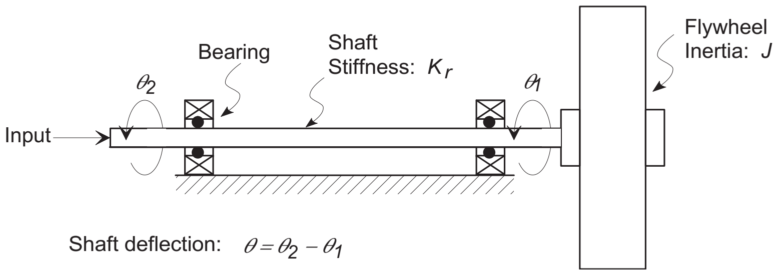
(d) If $R_2 = 22$ kΩ (22,000 Ω), $R_1 = 6.8$ kΩ, and $C = 0.47$ μFd (these are common values), find (i) The time-constant, and (ii) the steady-state output voltage v_{out} when the input is 2.5 v.

Problem 4: Nise Ch, 2, Prob. 30. For each system:

(a) Draw the linear graph representation of the system.

(b) Write the equations of motion (as Nise requests) in the Laplace domain using the node equations. (Do not solve.)

Problem 5: Consider the rotational system shown below:



The torsional stiffness of a cylindrical shaft of diameter D and length l is

$$K_r = G\frac{\pi}{32}\frac{D^4}{l}$$

where G is the shear modulus of the shaft material. A steel shaft, 5 m long and 5 cm in diameter, drives a steel cylindrical flywheel with a 30 cm diameter and a thickness of 5 cm. Steel has a density of $\rho = 7.8$ gm/cm^3 and a shear modulus of $G = 83$ GPa. Assume that each of the bearings exhibits a rotational viscous frictional coefficient $B = 0.1$ N-m-s/rad. The input is an angular velocity source $\Omega_s(t)$.

As a controls engineer you have been asked to design a closed-loop controller to maintain the flywheel at a constant speed under varying load conditions. In order to do this you need a transfer function model of the plant:

(a) What are the values of the shaft stiffness K_r and the flywheel moment of inertia J.

(b) Derive the transfer function (with numerical coefficients) relating the flywheel angular velocity Ω_1 to the input $\Omega_s(t)$.