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2.004 Dynamics and Control II
Spring 2008

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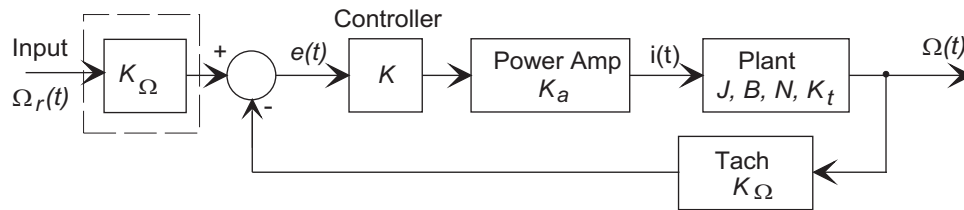
2.004 *Dynamics and Control II*
Spring Term 2008

Solution of Problem Set 4

Assigned: Feb. 29, 2008

Due: March 7, 2008

Problem 1:



- (a) Note that in the LAB setup tachometer is connected to the flywheel and consequently we assume that Ω represents the flywheel angular velocity. Angular velocity and torque of flywheel and motor (represented by "m") are related via gear ratio N:

$$\begin{cases} \Omega = \frac{1}{N}\Omega_m \\ T = N T_m \end{cases}$$

By using the above relation we can relate the flywheel dynamics ($J\dot{\Omega} + B\Omega = T$, ignoring the Coulomb friction) to the motor current, and then we derive the plant transfer function named $G(s)$:

$$J\dot{\Omega} + B\Omega = N T_m = N i(t) K_t \Rightarrow G(s) = \frac{\Omega(s)}{I(s)} = \frac{N K_t}{J s + B}$$

The system *practical* input is a voltage (equal to $\Omega(t)K_\Omega$) and that's why K_Ω block has been included in the input path. The transfer function between this voltage and output can be derived as:

$$\frac{\Omega(s)}{\Omega_r(s)K_\Omega} = \frac{K K_a G(s)}{1 + K_\Omega K K_a G(s)} = \frac{N K_t K K_a}{J s + B + N K_t K K_a K_\Omega}$$

- (b) Recall that $J = 3.0 \times 10^{-2} \text{ kg.m}^2$ (PSET 1) and $K_t = 30.2 \times 10^{-3} \frac{\text{N.m}}{\text{amps}}$ (PSET 2). Also $N = \frac{180}{44} = 4.09$ and $K_\Omega = 0.016 \frac{\text{v}}{\text{rpm}} = 0.016 \frac{\text{v}}{2\pi \text{ rad/s}} = 0.0025 \frac{\text{v.s}}{\text{rad}}$

$$\frac{\Omega(s)}{\Omega_r(s)} = \frac{6.3 \times 10^{-4} K}{3.0 \times 10^{-2} s + 0.014 + 6.3 \times 10^{-4} K}$$

$$K = 3 \Rightarrow \frac{\Omega(s)}{\Omega_r(s)} = \frac{0.0019}{3.0 \times 10^{-2} s + 0.0159} = \frac{0.19}{3s + 1.59}$$

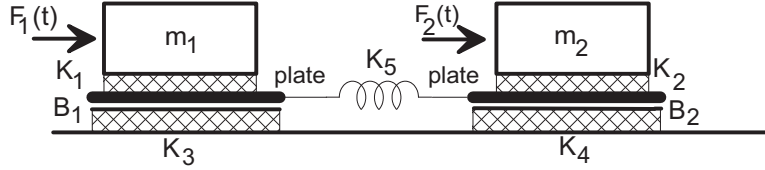
$$\tau = \frac{J}{B + N K_t K K_a K_\Omega} = \frac{3.0 \times 10^{-2}}{0.014 + 6.3 \times 10^{-4} K}$$

(c)

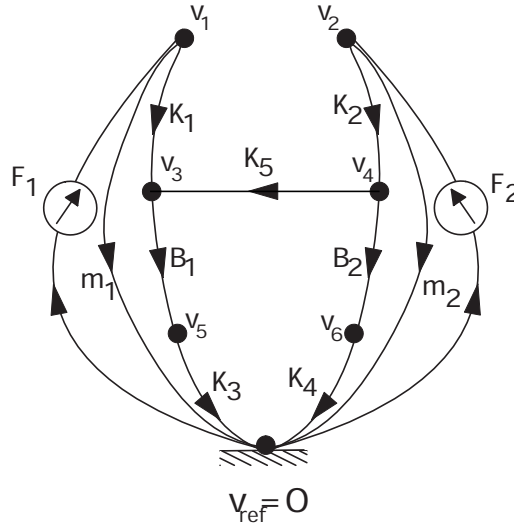
$$\frac{\Omega_{s.s.}}{\Omega_r} = \frac{NK_tKK_aK_\Omega}{B + NK_tKK_aK_\Omega}$$

$$\frac{\Omega_r - \Omega_{s.s.}}{\Omega_r} = \frac{B}{B + NK_tKK_aK_\Omega} = \frac{1}{1 + 0.0449K}$$

Problem 2:



(a) Note that this linear graph can be simplified by removing v_5 and v_6 nodes via using equivalent series impedances ($Z_{B_1} + Z_{K_3}$ and $Z_{B_2} + Z_{K_4}$).



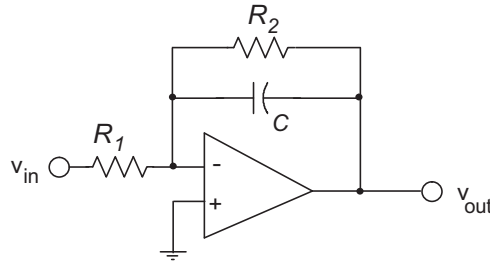
(b)

$$\left\{ \begin{array}{l} \text{node (1)} : F_{m_1} + F_{K_1} - F_1 = 0 \\ \text{node (2)} : F_{m_2} + F_{K_2} - F_2 = 0 \\ \text{node (3)} : F_{B_1} - F_{K_1} - F_{K_5} = 0 \\ \text{node (4)} : F_{B_2} - F_{K_2} + F_{K_5} = 0 \\ \text{node (5)} : -F_{B_1} + F_{K_3} = 0 \\ \text{node (6)} : -F_{B_2} + F_{K_4} = 0 \end{array} \right.$$

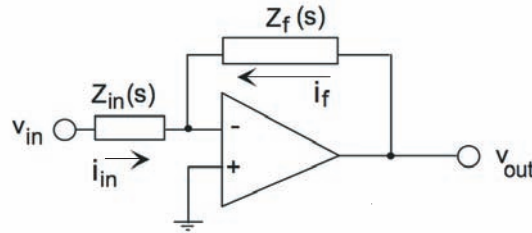
(c)

$$\left\{ \begin{array}{l} \text{node (1) :} \quad Y_{m_1}(v_1 - 0) + Y_{K_1}(v_1 - v_3) = F_1 \\ \text{node (2) :} \quad Y_{m_2}(v_2 - 0) + Y_{K_2}(v_2 - v_4) = F_2 \\ \text{node (3) :} \quad Y_{B_1}(v_3 - v_5) - Y_{K_1}(v_1 - v_3) - Y_{K_5}(v_4 - v_3) = 0 \\ \text{node (4) :} \quad Y_{B_2}(v_4 - v_6) - Y_{K_2}(v_2 - v_4) + Y_{K_5}(v_4 - v_3) = 0 \\ \text{node (5) :} \quad -Y_{B_1}(v_3 - v_5) + Y_{K_3}(v_5 - 0) = 0 \\ \text{node (6) :} \quad -Y_{B_2}(v_4 - v_6) + Y_{K_4}(v_6 - 0) = 0 \end{array} \right.$$

Problem 3:



(a) The generalized form of the circuit is depicted in the below figure:



The current entering negative terminal of op-amp is zero. So we can write the KCL for that node as:

$$i_{in} + i_f = 0$$

$$\frac{1}{Z_{in}}(V_{in} - V_-) + \frac{1}{Z_f}(V_{out} - V_-) = 0$$

Now note that $V_- = V_+ = 0$ so:

$$\frac{1}{Z_{in}}(V_{in} - 0) + \frac{1}{Z_f}(V_{out} - 0) = 0 \Rightarrow V_{out}(s) = -\frac{Z_f}{Z_{in}}V_{in}(s)$$

(b)

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_f}{Z_{in}} = -\frac{R_2 || \frac{1}{Cs}}{R_1} = -\frac{R_2}{R_1(R_2Cs + 1)}$$

(c)

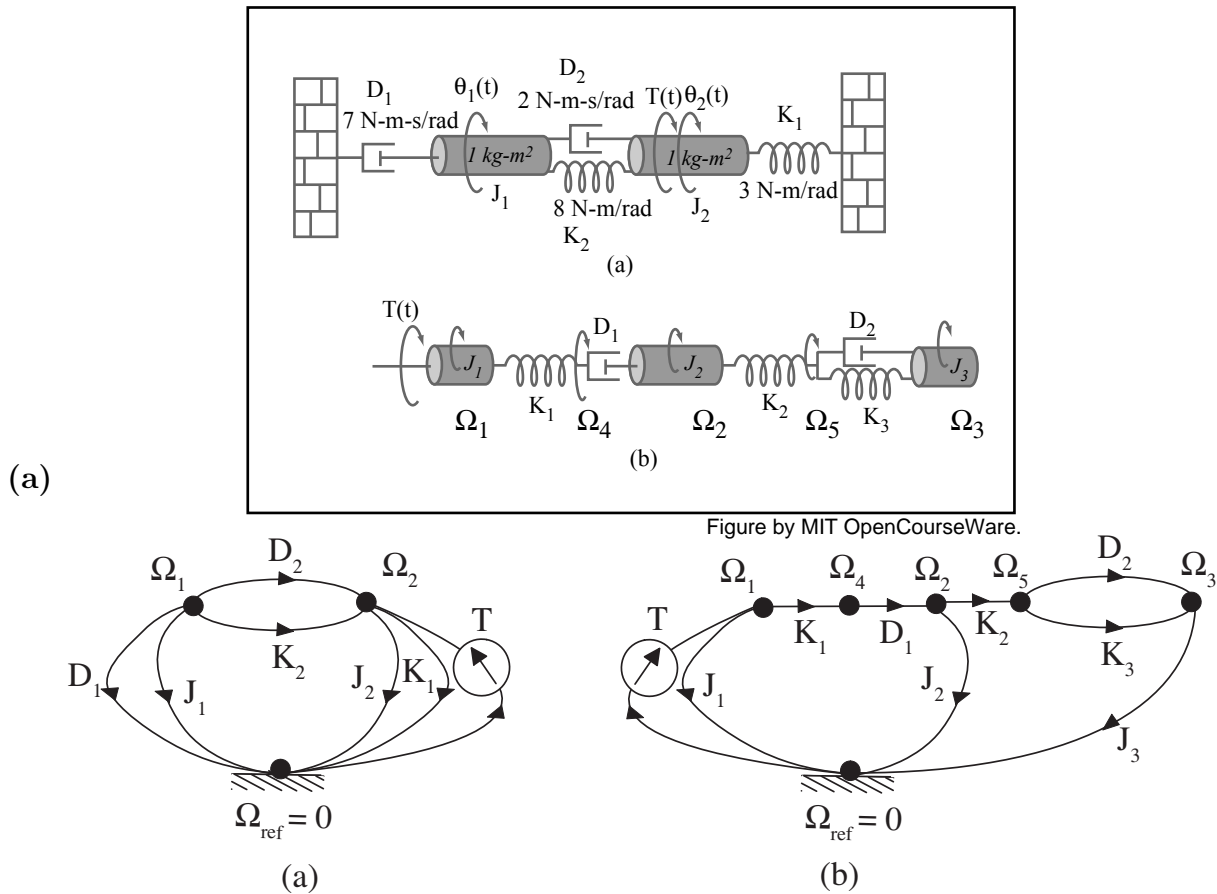
$$V_{out}(s)(R_2Cs + 1) = -V_{in}(s)\frac{R_2}{R_1} \Rightarrow R_2C\frac{dv_{out}(t)}{dt} + v_{out}(t) = -\frac{R_2}{R_1}v_{in}(t)$$

(d) (i) $\tau = R_2C = 10.3 \text{ ms}$

(ii) Steady State $\Rightarrow \frac{dv_{out}}{dt} = 0 \Rightarrow v_{out,s.s.} = -\frac{R_2}{R_1}v_{in,s.s.} = -8.1 \text{ v}$

Problem 4: Nise Ch. 2, Prob. 30.

New parameters and angular velocities are shown by bold fonts in the below figure. Furthermore, we introduce $\Omega_1 = \dot{\theta}_1$ and $\Omega_2 = \dot{\theta}_2$ for part (a).



(b)

Problem (a): Note that wherever we have $Y_? + Y_?$ we are dealing with parallel impedances.

$$\begin{cases} \text{node (1):} & T_{D_1} + T_{J_1} + T_{D_2} + T_{K_2} = 0 \\ \text{node (2):} & T_{K_1} + T_{J_2} - T_{D_2} - T_{K_2} = T \end{cases}$$

$$\begin{cases} \text{node (1):} & (Y_{D_1} + Y_{J_1})(\Omega_1 - 0) + (Y_{D_2} + Y_{K_2})(\Omega_1 - \Omega_2) = 0 \\ \text{node (2):} & (Y_{K_1} + Y_{J_2})(\Omega_2 - 0) - (Y_{D_2} + Y_{K_2})(\Omega_1 - \Omega_2) = T \end{cases}$$

$$\begin{cases} \text{node (1):} & (7 + 1s)(\Omega_1 - 0) + (2 + \frac{8}{s})(\Omega_1 - \Omega_2) = 0 \\ \text{node (2):} & (\frac{3}{s} + 1s)(\Omega_2 - 0) - (2 + \frac{8}{s})(\Omega_1 - \Omega_2) = T \end{cases}$$

$$\begin{cases} \text{node (1):} & (7 + 1s + 2 + \frac{8}{s})\Omega_1 - (2 + \frac{8}{s})\Omega_2 = 0 \\ \text{node (2):} & -(2 + \frac{8}{s})\Omega_1 + (\frac{3}{s} + 1s + 2 + \frac{8}{s})\Omega_2 = T \end{cases}$$

$$\begin{cases} \text{node (1):} & (s + 9 + \frac{8}{s})\Omega_1 - (2 + \frac{8}{s})\Omega_2 = 0 \\ \text{node (2):} & -(2 + \frac{8}{s})\Omega_1 + (s + 2 + \frac{11}{s})\Omega_2 = T \end{cases}$$

Problem (b): Note that Ω_4 and Ω_5 nodes could be eliminated in advance by parallel/series simplification.

$$\begin{cases} \text{node (1):} & T_{K_1} + T_{J_1} = T \\ \text{node (2):} & -T_{D_1} + T_{J_2} + T_{K_2} = 0 \\ \text{node (3):} & -T_{D_2} + T_{J_3} - T_{K_3} = 0 \\ \text{node (4):} & T_{D_1} - T_{K_1} = 0 \\ \text{node (5):} & T_{D_2} + T_{K_3} - T_{K_2} = 0 \end{cases}$$

$$\begin{cases} \text{node (1):} & Y_{K_1}(\Omega_1 - \Omega_4) + Y_{J_1}(\Omega_1 - 0) = T \\ \text{node (2):} & -Y_{D_1}(\Omega_4 - \Omega_2) + Y_{J_2}(\Omega_2 - 0) + Y_{K_2}(\Omega_2 - \Omega_5) = 0 \\ \text{node (3):} & -(Y_{D_2} + Y_{K_3})(\Omega_5 - \Omega_3) + Y_{J_3}(\Omega_3 - 0) = 0 \\ \text{node (4):} & Y_{D_1}(\Omega_4 - \Omega_2) - Y_{K_1}(\Omega_1 - \Omega_4) = 0 \\ \text{node (5):} & (Y_{D_2} + Y_{K_3})(\Omega_5 - \Omega_3) - Y_{K_2}(\Omega_2 - \Omega_5) = 0 \end{cases}$$

$$\begin{cases} \text{node (1):} & \frac{K_1}{s}(\Omega_1 - \Omega_4) + J_1s(\Omega_1 - 0) = T \\ \text{node (2):} & -D_1(\Omega_4 - \Omega_2) + J_2s(\Omega_2 - 0) + \frac{K_2}{s}(\Omega_2 - \Omega_5) = 0 \\ \text{node (3):} & -(D_2 + \frac{K_3}{s})(\Omega_5 - \Omega_3) + J_3s(\Omega_3 - 0) = 0 \\ \text{node (4):} & D_1(\Omega_4 - \Omega_2) - \frac{K_1}{s}(\Omega_1 - \Omega_4) = 0 \\ \text{node (5):} & (D_2 + \frac{K_3}{s})(\Omega_5 - \Omega_3) - \frac{K_2}{s}(\Omega_2 - \Omega_5) = 0 \end{cases}$$

Problem 5: This problem is moved to the next PSET (PSET 5).