2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

2.004 *Dynamics and Control II*

Spring Term 2008

Lecture 22[1]

Reading:

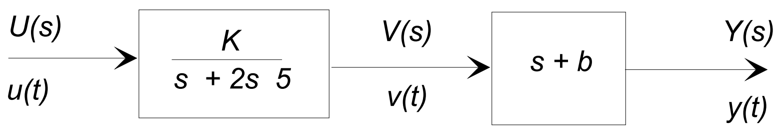
- Nise: 4.1 – 4.8

1 The Time-Domain Response of Systems with Finite Zeros

Consider a system:

$$Gs = \frac{K(s+b)}{s^2+2s+5},$$

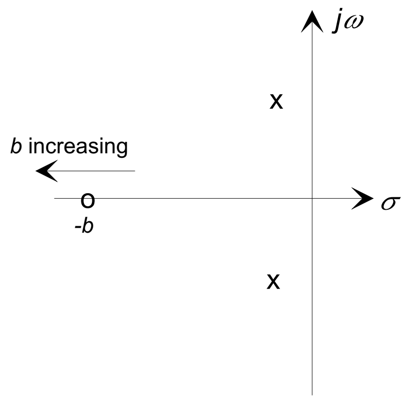
we have seen that we can consider this as two *cascade* blocks



Then if the response of the a system $1/D(s)$ is $v(t)$, then

$$y(t) = \frac{dv}{dt} + bv(t)$$

and as the zero (at $s = -b$) moves deeper into the l.h. s-plane,, the *relative* contribution of the derivative term decreases



and the system response tends toward a scaled version of the all pole response $v(t)$.

In general, the presence of the derivative terms in the response means that:

- The response is faster (shorter peak-time T_P and rise-time T_R).
- Greater overshoot in the response (if any). A zero may cause overshoot in the response of an over-damped second-order system.

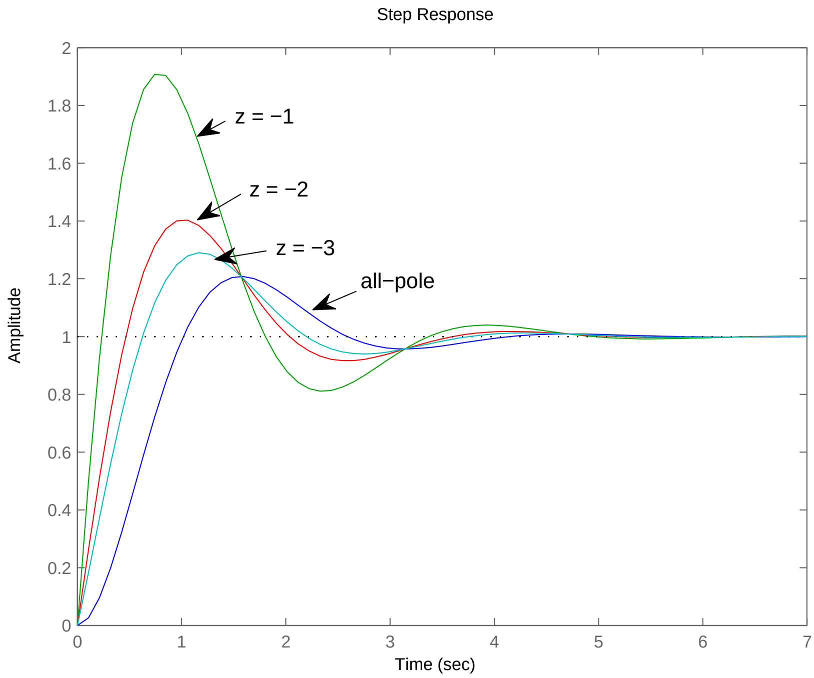
■ Example 1

The following MATLAB step response compares the response for the under-damped system

$$G(s) = \frac{5}{s^2 + 2s + 5}$$

with similar unity-gain systems with zeros at $s = -1, -2, -3$:

$$G(s) = \frac{5(s+1)}{s^2 + 2s + 5}, \quad G(s) = \frac{5/2(s+2)}{s^2 + 2s + 5}, \quad G(s) = \frac{5/3(s+3)}{s^2 + 2s + 5}$$



Note the increase in the overshoot, and the decrease in T_P as the zero approaches the origin.

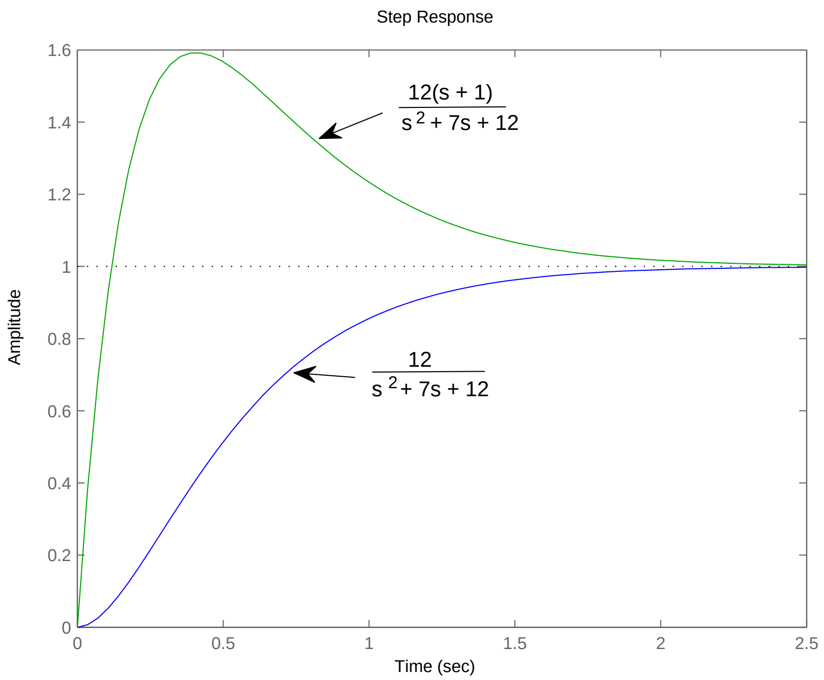
■ Example 2

The following MATLAB step response compares the response for the unity-gain overdamped system

$$G(s) = \frac{12}{s^2 + 7s + 12}$$

with two real poles at $s = -3$ and $s = -4$ with the similar system with a zeros at $s = -1$:

$$G(s) = \frac{12(s + 1)}{s^2 + 7s + 12}$$



Note the overshoot caused by the zero, but that the overshoot is not oscillatory. Clearly the rise-time T_R is much shorter for the system with the zero.

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2 The Time-Domain Response of Systems where the Order of the Numerator equals the Order of the Denominator

Consider systems of the form

$$G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

where the degree of the numerator equals that of the denominator. In such systems it is possible to do polynomial division and write the transfer function as

$$G(s) = \frac{N(s)}{D(s)} = K + \frac{N'(s)}{D(s)}$$

where $N'(s)$ is a polynomial of degree less than that of $D(s)$.

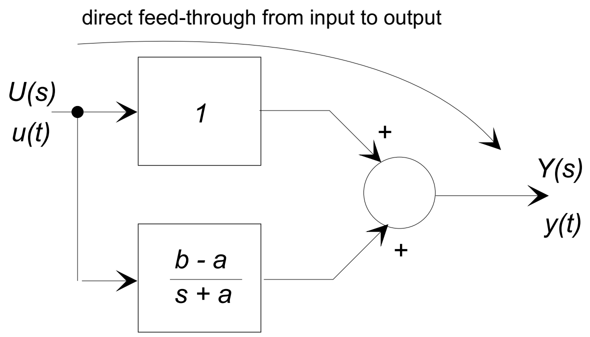
For example, a system with transfer function

$$G(s) = \frac{s+a}{s+b}$$

may be written

$$G(s) = 1 + \frac{b-a}{s+a},$$

which may be represented in block-diagram form



showing a direct *feed-through* of the input into the output. In other words, when the order of the numerator is the same of the denominator the *input will appear directly as a component of the output.*

The step-response $y_{step}(t)$ of this system will therefore be

$$y_{step}(t) = u_s(t) + \frac{b-a}{a}\left(1 - e^{-at}\right)$$

where $u_s(t)$ is the unit-step (Heaviside) function.

Note:

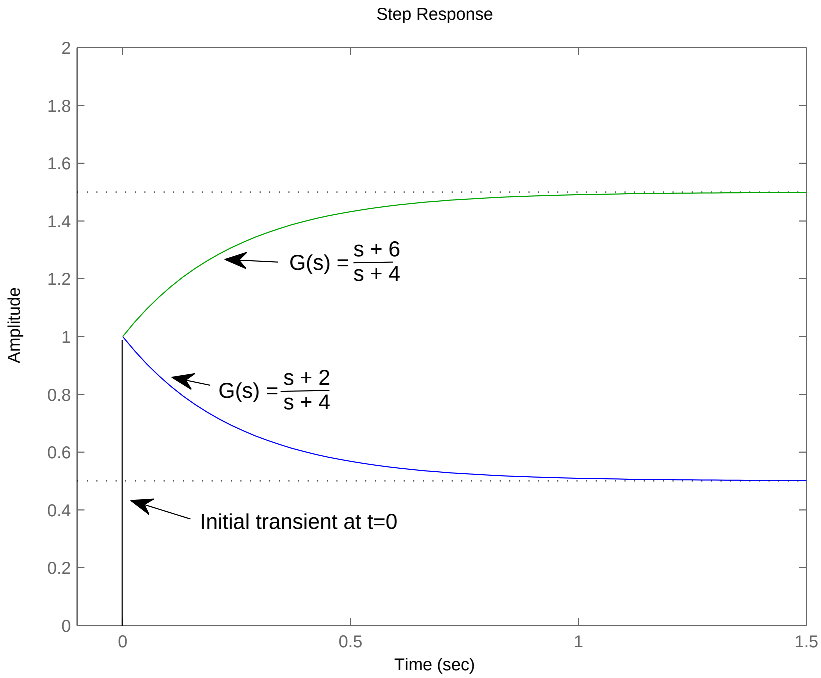
- That $y_{step}(0^+) = 1$, that is there is a step transient in the response (which does not occur if the order of $N(s)$ is less than that of $D(s)$).
- The steady-state step response $y_{ss} = b/a$, and if $b > a$ then $y_{ss} > 1$, while if $a > b$ $y_{ss} < 1$.

The following MATLAB plot shows the step responses for the two systems

$$G(s) = \frac{s+6}{s+4} \quad (b > a), \qquad \text{and} \quad G(s) = \frac{s+2}{s+4} \quad (a > b)$$

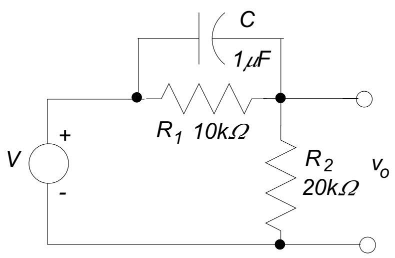
with step responses

$$y_{step}(t) = 1 + \frac{2}{4}\left(1 - e^{-4t}\right) \quad \text{and} \quad y_{step}(t) = 1 - \frac{2}{4}\left(1 - e^{-4t}\right)$$



■ Example 3

Find the step response of the following electrical circuit:



The transfer function is

$$\boxed{G(s) = \frac{V_o(s)}{V(s)} = \frac{s + 1/R_1C}{s + (R_1 + R_2)/R_1R_2C}}$$

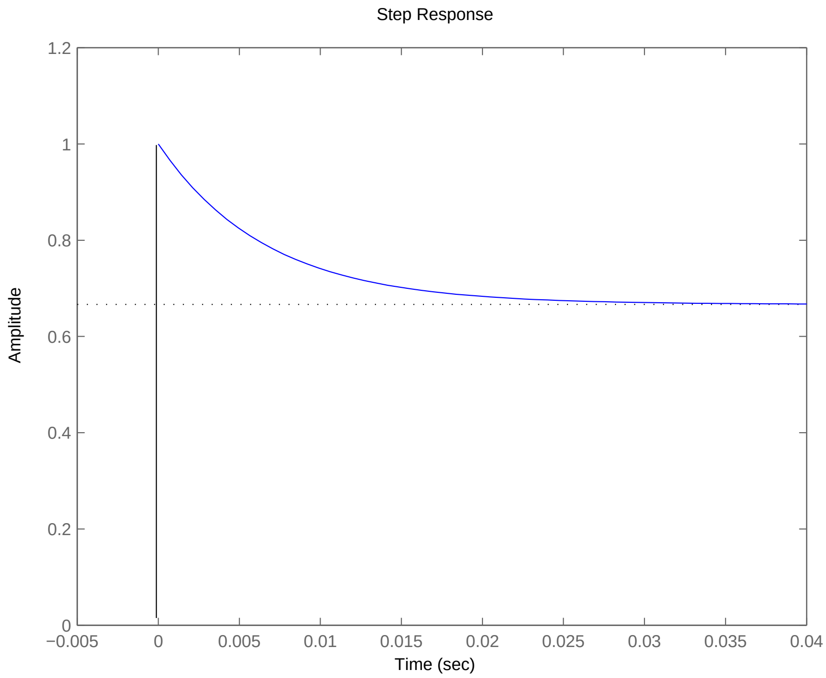
and with the values shown

$$G(s) = \frac{V_o(s)}{V(s)} = \frac{s + 100}{s + 150} = 1 - \frac{50}{s + 150}.$$

The step response is therefore

$$y_{step} = 1 - \frac{50}{150}\left(1 - e^{-150t}\right) = \frac{2}{3} + \frac{1}{3}e^{-150t}$$

which is plotted below:



■ Example 4

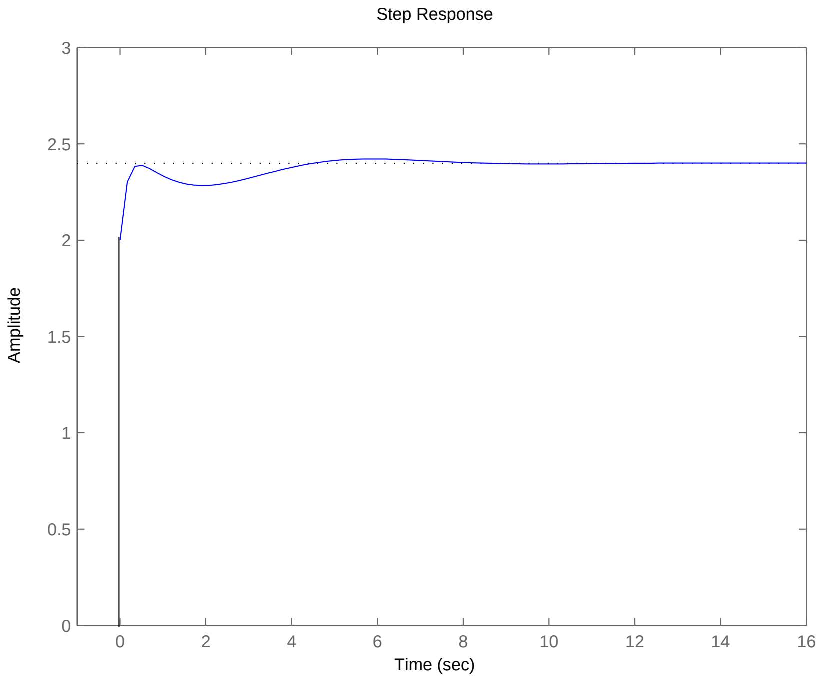
Find the step response of the following third-order system:

$$\begin{aligned} G(s) &= \frac{2s^3 + 17s^2 + 13s + 12}{s^3 + 7s^2 + 6s + 5} \\ &= 2 + \frac{3s^2 + s + 2}{s^3 + 7s^2 + 6s + 5} \end{aligned}$$

showing a direct feed-through term of amplitude two. From Maple-Syrep, the step response is

$$y_{step}(t) = 2.4 - 0.5307e^{-6.157t} + 0.1307e^{-0.4213t}\cos(0.7966t) - 0.2667e^{-0.4213t}\sin(0.7966t)$$

from which $y_{step}(o^+) = 2$, and $y_{ss} = 2.4$. The step response is plotted below.


Step Response
Amplitude
Time (sec)
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16