13.002
Introduction to Numerical Methods for Engineers
Take Home Exam
Issued: Thursday, Mar. 10, 2005
Due: Friday, Mar. 18, 2005

Problem 1.

$$\mathbf{A} = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}$$

a) Compute the inverse $\mathbf{A}^{-1}$ and determine the solution to $\mathbf{Ax} = \mathbf{b}$ when $\mathbf{b} = (\,4\ 3\ 3\ 1\,)^T$.

b) Assume that the right-hand side $\mathbf{b}$ is perturbed by a vector $\delta\mathbf{b}$ such that $\|\delta\mathbf{b}\|_\infty \leq 0.01$ Give an upper bound for $\|\delta\mathbf{x}\|_\infty$, where $\delta\mathbf{x}$ is the corresponding perturbation in the solution.

c) Compute the condition number $\kappa(A)$ and compare it with the bound for the quotient between $\frac{\|\delta\mathbf{x}\|}{\|\mathbf{x}\|}$ and $\frac{\|\delta\mathbf{b}\|}{\|\mathbf{b}\|}$ which can be derived from (b).

Problem 2.

a. Make algorithms for finding the roots of the equation

$$x\tan(x) = 2\,, \tag{1}$$

in the interval $x \in [0, \pi/2]$, using:

1. Newton-Ralphson Iteration
2. The Secant Method

b. Make a graph of the relative errors vs iteration step for the two algorithms and compare their convergence behavior to that of a *quadratic* convergence.