2.611 Fall 2006
Problem Set #5 Answer Key

Problem 1 (Just parts A and B).

Solution Exercise 1

A).

- The mechanical and heat losses are both approximately 10%, so mechanical and heat efficiency are both 0.9. Consequently the effective engine efficiency is:

$$\eta_e = 0.9 \cdot 0.9 \cdot 0.56 = 0.454 = 45.4\%$$

- To determine the mean effective pressure, first realise that work is converted heat, which is converted chemical energy in fuel. The amount of the chemical energy is determined by the available amount of air through the air-fuel ratio. The cylinder fills with air of absolute pressure 3+1=4 bar and temperature of 273+50=323 K.

  Consequently, theoretically the mass trapped in the volume is:

$$m = \frac{p \cdot V_s}{R \cdot T} = \frac{4 \cdot 10^5 \cdot V_s}{287 \cdot 323} = 4.315 \cdot V_s \text{ kg}$$

  The swept volume will not need to be determined numerically at this stage, because p_{me} is work over swept volume, so it will be eliminated later. As charging is 95% effective, the actual amount of fresh combustion air is less than the trapped mass as calculated theoretically.

$$m_{ca} = 0.95 \cdot m = 4.10 \cdot V_s \text{ kg}$$

  The air excess ratio is 2 and the stoichiometric air-fuel ratio is 14.5, so the air fuel ratio is 29. The amount of fuel consequently is:

$$m_f = {m_{ca}}/{29} = 0.141 \cdot V_s \text{ kg}$$

  The heat associated with this amount of fuel is:

$$Q_f = m_f \cdot 42700 = 6035 \cdot V_s \text{ kJ}$$

  The useful work output is less and can be determined with the effective efficiency as calculated under A).:

$$W_e = \eta_e \cdot Q_f = 2740 \cdot V_s \text{ kJ}$$

  The mean effective pressure is effective work divided by swept volume so:

$$p_{me} = 2740 \text{ kPa} = 27.4 \text{ bar}$$

- To determine the brake power, we first need to calculate some other parameters, such as the stroke length:

$$L_S = \frac{c_m}{(2 \cdot n_e / 60)} = 0.3 \text{ m}$$

- The bore diameter now follows from the stroke-bore ratio, which is 1.3 :

$$D_B = \frac{L_S}{1.3} = 0.231 \text{ m}$$

The swept volume can now be determined:

$$V_s = \frac{\pi}{4} \cdot D_B{}^2 \cdot L_s = 0.0126 \text{ m}^3$$

The work done by each cylinder is:

$$W_e = p_{me} \cdot V_s = 2740 \cdot 10^3 \cdot 0.0126 = 34.5 \cdot 10^3 \text{ J}$$

The cycle frequency for a 4-stroke 12 cylinder engine equals:

$$f = \frac{n_e \cdot 12}{60 \cdot k} = 100 \; Hz$$

So, the brake power is:

$$P_B = W_e \cdot f = 3450 \cdot 10^3 \; W = 3450 \; kW$$

B).

- Keeping the assumptions unaltered, the mechanical and heat efficiency remain both 0.9. Consequently the effective engine efficiency would be the same:

$$\eta_e = 0.9 \cdot 0.9 \cdot 0.56 = 0.454 = 45.4\%$$

- No turbocharging means that the charge pressure is lowered to 1 bar. The mass trapped theoretically in the swept volume then is:

$$m = \frac{p \cdot V_S}{R \cdot T} = \frac{1 \cdot 10^5}{287 \cdot 323} = 1.079 \cdot V_S$$

- This is of course a factor 4 less than in case A. This also applies to the actual amount of fresh combustion air and consequently, since air fuel ratio is assumed not to alter, also to the amount of fuel. But then heat and effective work will be smaller by a factor 4 too, as well as the mean effective pressure ratio:

$$p_{me} = \frac{2740}{4} = 685 \text{ kPa} = 6.85 \text{ bar}$$

- Assuming the same engine speed, power also is a factor 4 less than in case A):

$$P_B = \frac{3450 \cdot 10^3}{4} = 862 \cdot 10^3 \text{ W} = 862 \text{ kW}$$

C).

- Mechanical and heat losses: although power of the non turbocharged engine is considerable lower than the turbocharged engine it is not likely that the mechanical losses and heat losses will also be lower by a factor 4. In fact they have a tendency to remain the same in absolute sense. Consequently, the mechanical and heat losses as a percentage of ideal work and ideal heat input will be considerably higher than the assumed 10%. A view on Figure 7.40 would suggest 20% for the mechanical losses and 25% for the heat losses.

- Another question would be what peak pressure one would allow in the non-turbocharged engine. If it would be possible to keep the same peak pressure, for instance by increasing the compression ratio, the thermodynamic efficiency would be much better. But the compression ratio could become excessive and peak pressure for a non-turbocharged engine normally will be lower.
- The effect to the overall engine efficiency therefore is the result of the opposing effects of increased mechanical and heat losses and better thermodynamic efficiency. A view on Figure 7.43 would suggest that it would be 41% for a peak pressure of 120 bar instead of the 45% found in case A and used in case B, i.e. lower by a factor 1.1.
- The trapped air efficiency of a non-turbocharged engine will be worse than for a turbocharged engine since it is not possible to have an overpressure across the engine and hence scavenging is less effective. It could well be that the assumed 95% for the trapped air efficiency would drop to 80%, i.e. lower by a factor 1.20. This would translate directly into amount of fresh air, amount of fuel and heat input lower by the same factor. Combined with the somewhat lower engine efficiency, the effective work would be lower by the product of the two factors, i.e. 1.1 x 1.2 = 1.3. The mean effective pressure:

$$p_{me} = \frac{685}{1.3} = 526 \text{ kPa} = 5.3 \text{ bar}$$

- This is a typical value for diesel engines of the pre-turbocharger era. The engine power would be only:

$$P_B = \frac{862 \cdot 10^3}{1.3} = 663 \cdot 10^3 \text{ W} = 663 \text{ kW}$$

Problem 2

Diesel engine (two stroke) $V_1 = 1.0m^3$, $r_v = 13.0$, $r_p = 1.8$, $r_c = 1.4$, $T_1 = 300^o K$,
$p_1 = 1$ bar $\left(100kN/m^2\right)$, $\gamma = C_p / C_v = 1.00/0.7143$, $R = 0.286kJ/kgK$, 79 rpm

Temperature: $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300x13^{0.4} = 836.9^o K$$

$$T_3 = T_2 x r_p = 836.9x1.8 = 1506^o K$$

$$T_4 = T_3 x r_c = 1506.5x1.4 = 2109.1^o K$$

$$T_5 = T_4 x\left(V_4 / V_5\right) = 2109.1\left(\frac{1.4}{13.0}\right)^{0.4} = 864.9^o K$$

$Q_{H1} = mC_v\left(T_3 - T_2\right)$, where $m = p_1 V_1 / RT_1 = 100x1/\left(.286x300\right)$

b. $m = 1.1655kg$

c. $Q_{HI} = 1.1655x\frac{1}{1.4}\left(1506.5 - 836.9\right) = 557.4kJ$

$$Q_{H2} = mC_p\left(T_4 - T_3\right) = 1.1655x1.0\left(2109.1 - 1506.5\right) = 702.3kJ$$

$$Q_L = -mC_2\left(T_5 - T_1\right) = -1.1655x\frac{1}{1.4}\left(864.9 - 300\right) = -470.29kJ$$

d. Work output = $\dot{Q}_{HI} + \dot{Q}_{H2} - \dot{Q}_L = 557.4 + 702.3 - 470.29 = 789.45kJ$ per cycle

Power of the two stroke engine = Work $xrpm/60 = 1.039.4kW/cycle$

e. The turbocharger compressor pressure ratio $p_{out}/p_{in} = 3.9$, $\eta_{pc} = 0.83$

$$T_{out}/T_{in} = \left(p_{out}/p_{in}\right)^{\frac{\gamma-1}{\eta_{pc}\gamma}} = 3.9 = 1.5976$$

$$T_{out} = T_{comp} = 300x1.5976 = 479.3$$

$$\varepsilon = \frac{T_{comp} - T_1^1}{T_{comp} - T_w} = \frac{479.3 - T_1^1}{479.3 - 300}$$

$$T_1^1 = 479.3 - 0.8\left(479.3 - 300\right) = 335.8^o K$$

f. All temperatures are scaled from previous results by $335.8/300 = 1.1195$

$T_2^1 = 936.97^o K$ $\quad$ $T_3^1 = 1686.6^o K$

$T_4^1 = 2361.2^o K$ $\quad$ $T_5^1 = 968.3^o K$

Heat transfers Q_{H1}, Q_{H2}, and Q_L are all scaled by mass of air in cylinder and temperature.

$$Q_{H1}^1 = m^1 c_v \left(T_3^1 - T_2^1\right) = \frac{m^1}{m} x \frac{T_1^1}{T_1} x Q_{H1}$$

$$m^1 = p_1^1 V_1 / R T_1^1 = m x \frac{p_1^1}{p_1} x \frac{T_1}{T_1^1} = 4.06 kg$$

$$\therefore Q_{H1}^1 = Q_H x \frac{p_1^1}{p_1} = 3.9 x Q_m = 2173.9 kJ$$

$$Q_{H2} = 2739.1 kJ$$

$$Q_L^1 = -1834.1 kJ$$

The work per cycle is proportional to the heat transfers.

$\therefore$ Work output$^1 = 3078.9 kJ$ per stroke

and $Power^1 = 4053.8 kW$ per cylinder

Problem 3

Given:

$T_1 := 300K$ $\quad p_1 := 3bar$ $\quad \gamma := 1.4$ $\quad c_p := 1.00\frac{kJ}{kg \cdot K}$ $\quad LHV := 43000\frac{kJ}{kg}$

From the article:

$bore := 0.57m$ $\quad eng\_RPM := 375RPM$ $\quad r_v := 14$

$stroke := 0.75m$ $\quad piston\_speed := 9.4\frac{m}{s}$

a. Temperature calculations:

$\frac{T_2}{T_1} = r_v^{\gamma-1}$ so $T_2 := T_1 \cdot r_v^{\gamma-1}$ $\quad T_2 = 862.1K$

$T_3 := T_2 + 300K$ $\quad T_3 = 1162.1K$

$T_4 := T_3 + 800K$ $\quad T_4 = 1962.1K$

$r_p := \frac{T_3}{T_2}$ $\quad r_c := \frac{T_4}{T_3}$

$T_5 := T_1 \cdot r_c^{\gamma} \cdot r_p$ $\quad T_5 = 841.9K$

Volume and pressure calculations:

$\text{swept\_volume} := \frac{\pi}{4} \cdot \left(\text{bore}^2\right) \cdot \text{stroke}$ and $V_1 - V_2 = \text{swept\_volume}$

$r_v = \frac{V_1}{V_2}$ so

$V_1 = r_v \cdot V_2$

$r_v \cdot V_1 - V_1 = r_v \cdot V_1 - r_v \cdot V_2$

$V_1 \cdot \left(r_v - 1\right) = r_v \cdot \left(V_1 - V_2\right)$

$V_1 \cdot \left(1 - \frac{1}{r_v}\right) = V_1 - V_1 = \text{swept\_volume}$

$V_1 := \frac{\text{swept\_volume}}{1 - \frac{1}{r_v}}$ $\quad V_1 = 206.104L$

$V_2 := \frac{V_1}{r_v}$ $\quad V_2 = 14.722L$

$p_2 := p_1 \cdot r_v^{\gamma}$ $\quad p_2 = 120.7bar$

$p_3 := p_2 \cdot r_p$ $\quad p_3 = 162.7bar$

$p_4 := p_3$ $\quad p_4 = 162.7bar$

$p_5 := p_1 \cdot \left(\frac{T_5}{T_1}\right)$ $\quad p_5 = 8.4bar$

b). $r_p = 1.348$

$r_c = 1.688$

c). the thermal efficiency

$\eta_{th\_dual} := 1 - \frac{1}{r_v^{\gamma-1}} \cdot \left[\frac{r_p \cdot r_c^{\gamma} - 1}{\left(r_p - 1\right) + \gamma \cdot r_p \cdot \left(r_c - 1\right)}\right]$ $\quad \eta_{th\_dual} = 0.618$

$sfc := \frac{3600\frac{s}{hr}}{\eta_{th\_dual} \cdot LHV}$ $\quad sfc = 0.135\frac{kg}{kW \cdot hr}$

d). mass of air in each cylinder--use ideal gas law

$PV = mRT$ (at condition 1, for example)

$$m := \frac{p_1 \cdot V_1}{\left(287 \frac{J}{kg \cdot K}\right) \cdot T_1}$$

$m = 0.718kg$

e) power output per cylinder

$c_p = \gamma \cdot c_v$ so $c_v := \frac{c_p}{\gamma}$

$$m\_dot := m \cdot 375 \frac{rev}{min} \cdot \left(\frac{min}{60sec}\right) \cdot \left(\frac{1power\_stroke}{2strokes}\right)$$

$$Q\_dot_{H1} := m\_dot \cdot c_v \cdot T_2 \cdot (r_p - 1)$$

$$Q\_dot_{H2} := m\_dot \cdot \gamma \cdot c_v \cdot T_2 \cdot r_p \cdot (r_c - 1)$$

$$Q\_dot_L := -m\_dot \cdot c_v \cdot T_1 \cdot \left(r_c^{\gamma} \cdot r_p - 1\right)$$

$$W\_dot := Q\_dot_{H1} + Q\_dot_{H2} + Q\_dot_L$$

$W\_dot = 1407.6kW$ per cyl

f). mean effective pressure

$$MEP = \frac{\frac{power}{cyl}}{\frac{swept\_volume}{sec}}$$ so $$MEP := \frac{W\_dot}{swept\_volume \cdot \left[375 \frac{rev}{min} \cdot \left(\frac{min}{60 \cdot sec}\right) \cdot \left(\frac{1 \cdot power\_stroke}{2 \cdot strokes}\right)\right]}$$

$MEP = 23.53bar$

g). Include a comparison of your calculated results with the numbers in the article.