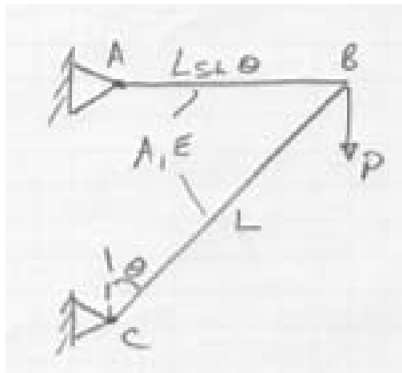
2.001 - MECHANICS AND MATERIALS I
Lecture #10
10/16/2006
Prof. Carol Livermore

From last time:



1. Equilibrium

$$F_{BC} = -P/\cos\theta$$

$$F_{AB} = P\tan\theta$$

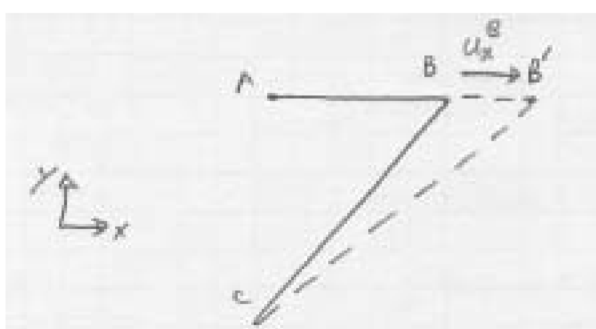
2. Force deformation relationships ("$P = k\delta$")

$$\delta^{AB} = \frac{PL\tan\theta\sin\theta}{AE}$$

$$\delta^{BC} = \frac{-PL}{AE\cos\theta}$$

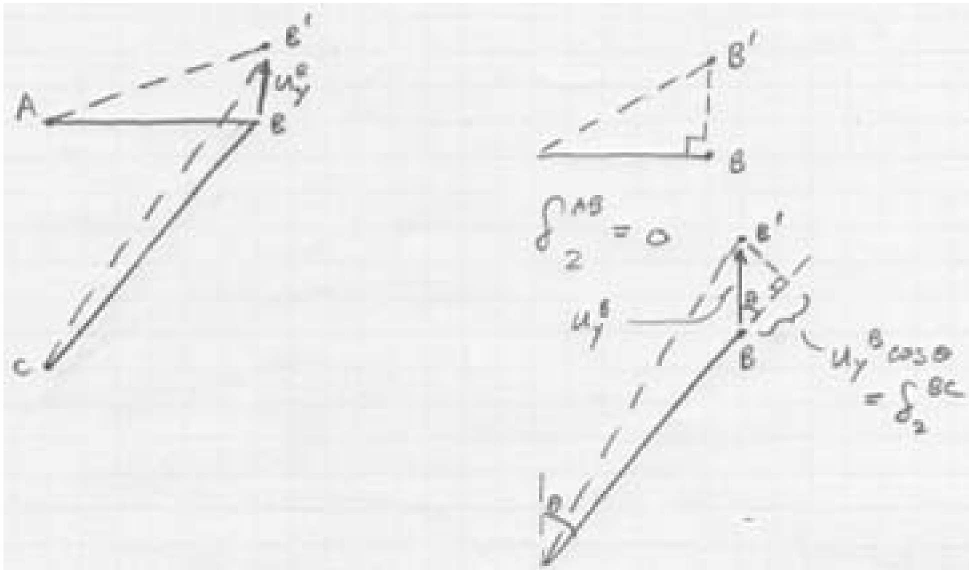
3. Compatibility

a. u_x^B



$$\delta_1^{AB} = u_x^B$$
$$\delta_1^{BC} = u_x^B \sin\theta$$

b. u_y^B



$$\delta_2^{AB} = 0$$
$$\delta_2^{BC} = u_x^B \cos\theta$$

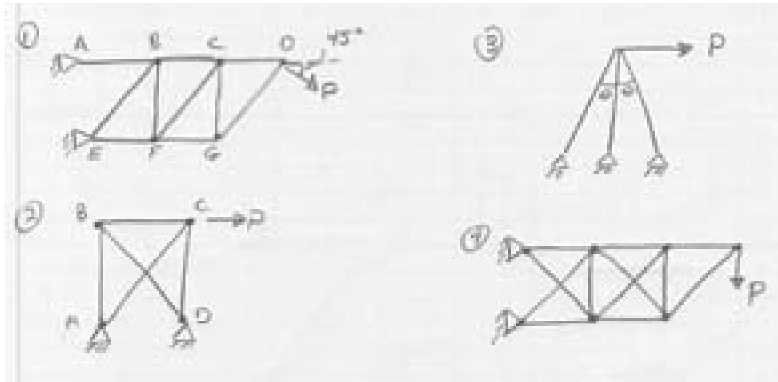
Sum:

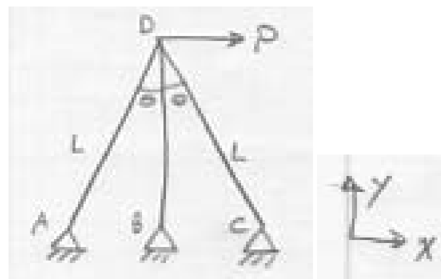
$$\begin{aligned} \delta^{AB} &= \delta_1^{AB} + \delta_2^{AB} \\ &= u_x^B + 0 = u_x^B = \delta^{AB} \end{aligned}$$

$$\begin{aligned} \delta^{BC} &= \delta_1^{BC} + \delta_2^{BC} \\ &= u_x^B \sin\theta + 0 = u_y^B \cos\theta = \delta^{BC} \end{aligned}$$

$$u_x^B = \frac{PL\tan\theta\sin\theta}{AE}$$
$$u_y^B = \frac{\delta^{BC}}{\cos\theta} - \frac{u_x^B \sin\theta}{\cos\theta}$$
$$u_y^B = \frac{-PL}{AE\cos^2\theta} - \frac{PL\tan\theta\sin^2\theta}{AE\cos\theta}$$

2, 3, 4 are statically indeterminate! (more bars than are needed to support the structure)



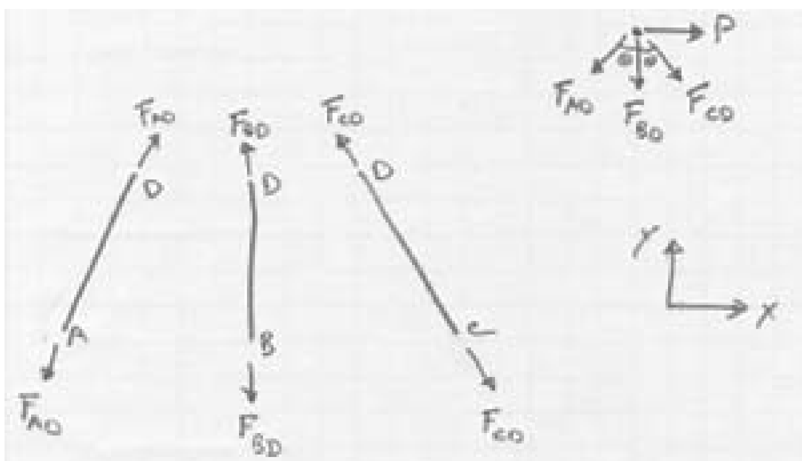


A, E for all bars
Forces? Deformations? Displacement of point D?

Unconstrained D.O.F

1. u_x^D
2. u_y^D

Equilibrium



$$\sum F_{D_x} = 0$$

$$P + F_{DC} \sin\theta - F_{AD} \sin\theta = 0$$

$$\sum F_{D_y} = 0$$

$$-F_{DC} \cos\theta - F_{AD} \cos\theta - F_{BD} = 0$$

Force-Deformation Relationships

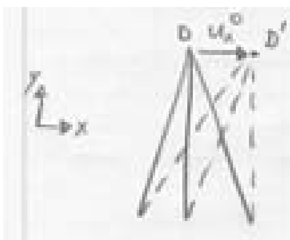
$$F_{AD} = k_{AD}\delta^{AD} = \frac{AE}{L}\delta^{AD}$$

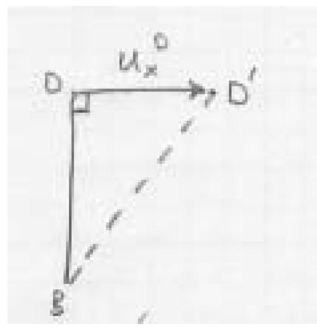
$$F_{CD} = k_{CD}\delta^{CD} = \frac{AE}{L}\delta^{CD}$$

$$F_{BD} = k_{BD}\delta^{BD} = \frac{AE}{L\cos\theta}\delta^{BD}$$

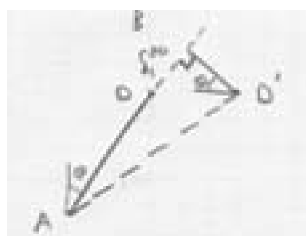
Compatibility

u_x^D

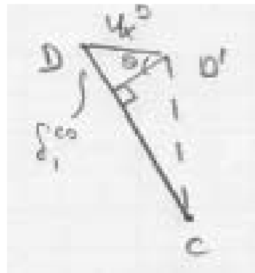




$$\delta_1^{BD} = 0$$

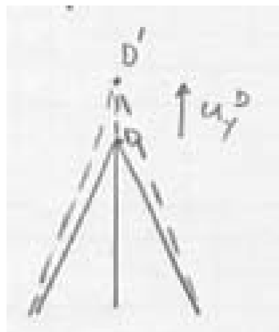


$$\delta_1^{AD} = u_x^D \sin\theta$$

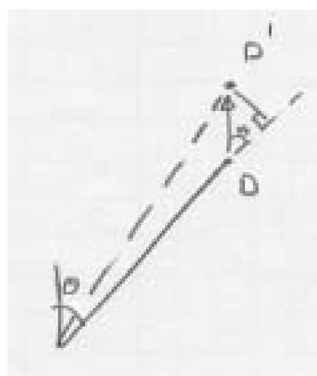


$$\delta_1^{CD} = u_x^D \sin\theta$$

u_y^D



$$\delta_2^{BD} = u_y^D$$



$$\delta_2^{AD} = u_y^D \cos\theta$$
$$\delta_2^{CD} = u_y^D \cos\theta$$

Sum:

$$\delta^{AD} = u_x^D \sin\theta + u_y^D \cos\theta$$
$$\delta^{BD} = 0 + u_y^D = u_y^D$$
$$\delta^{CD} = -u_x^D \sin\theta + u_y^D \cos\theta$$

Substitute δ into F

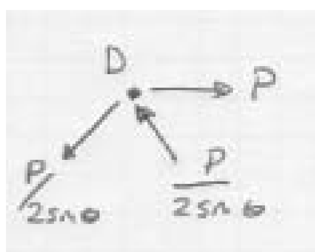
$$F_{AD} = \frac{AE}{L}(u_x^D \sin\theta + u_y^D \cos\theta)$$

$$F_{BD} = \frac{AE}{L\cos\theta} u_y^D$$
$$F_{CD} = \frac{AE}{L}(-u_x^D \sin\theta + u_y^D \cos\theta)$$

Substitue F into equilibrium

$$P + F_{CD}\sin\theta - F_{AD}\sin\theta = 0$$
$$P + \frac{AE}{L}(-u_x^D \sin\theta + u_y^D \cos\theta)\sin\theta - \frac{AE}{L}(u_x^D \sin\theta + u_y^D \cos\theta)\sin\theta = 0$$
$$u_y^D \cos\theta \rightarrow 0$$
$$P - \frac{2AE}{L} u_x^D \sin^2\theta = 0$$
$$u_x^D = \frac{PL}{2AE\sin^2\theta}$$

$$F_{DC}\cos\theta + F_{AD}\cos\theta + F_{BD} = 0$$
$$\frac{AE}{L}(u_y^D \cos\theta - u_x^D \sin\theta)\cos\theta + \frac{AE}{L}(u_x^D \sin\theta + u_y^D \cos\theta)\cos\theta + \frac{AE}{L\cos\theta} u_y^D = 0$$
$$\frac{AE}{L}\left[2u_y^D \cos^2\theta + \frac{u_y^D}{\cos\theta}\right] = 0$$
$$\frac{AE}{L} u_y^D \left[2\cos^2\theta + \frac{1}{\cos\theta}\right] = 0$$



$$u_y^D = 0$$

$$F_{AD} = \frac{P}{2\sin\theta}$$
$$F_{BD} = 0$$
$$F_{CD} = \frac{-P}{2\sin\theta}$$