2.002 MECHANICS & MATERIALS II

INTRODUCTION TO LINEAR ELASTIC FRACTURE MECHANICS

- Sometimes, components and structures which have been properly designed to avoid failures due to exe-cessive elastic deflection or plastic yielding, will fail into two or more pieces in a catastrophic brittle manner.

- This usually occurs due to the presence of cracks in the structures. These cracks can extend rapidly at applied load levels which are nominally well within the linear load-displacement response of the component.

- Over the last 50 years a methodology based on the theory of linear elastic fracture mechanics (LEFM) has been developed to help design components and structures against this catastophic mode of failure.

- The essence of the theory is embodied in the following **fracture initiation criterion** for globally brittle fracture of structures:

$$\boxed{K_I \leq K_{Ic}}$$

  The quantity K_1 is called the mode I **stress intensity factor**. It is a function of the applied stress, the crack length and the geometry of the body. The quantity K_{Ic} is a **critical stress intensity factor** or **fracture toughness**. It a material property which measures the resistance of a material to the propagation of a crack.

- We shall outline the development of this fracture criterion, the conditions under which it is applicable, and discuss how the theory is typically applied in practice.

IMPORTANCE OF FRACTURE MECHANICS

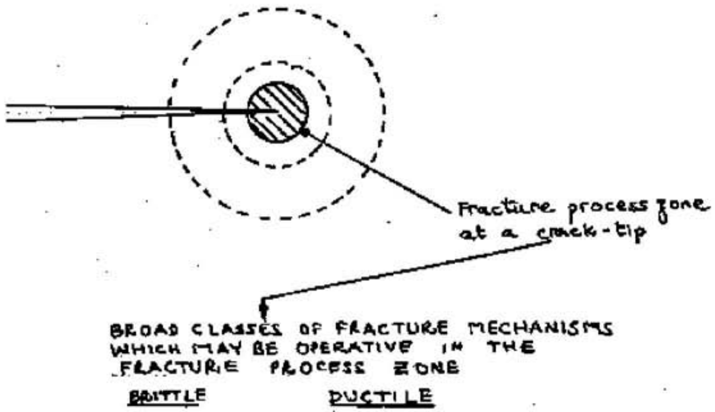
- **Most engineering structures either initially contain small crack–like defects, or they readily develop such defects during service.**

- The influence of pre-existing cracks on the strength of materials need to be understood and quantified.

- The initiation and growth of crack-like defects during service — fatigue-cracks, needs to be understood and quantified.

- A defect-tolerant design and maintenance philosophy needs to be developed.

- **Local**, or microscale, characterization of the mechanisms of fracture as brittle or ductile:



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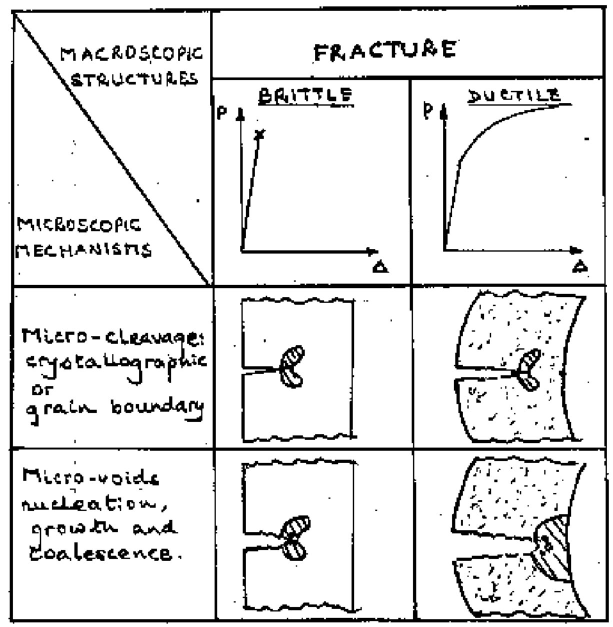
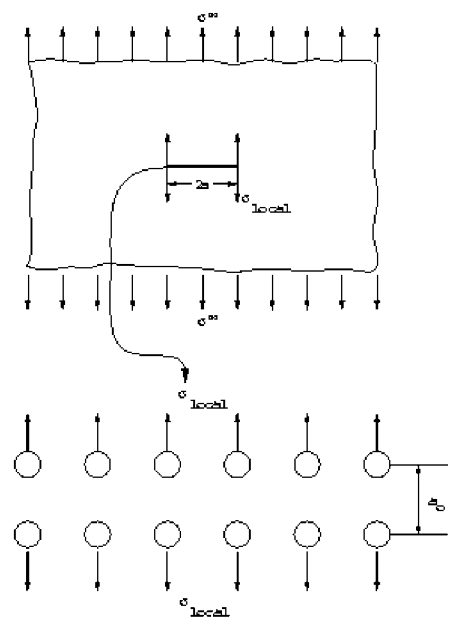
- **Global**, or macroscopic, characterization of the fracture of a structure as brittle or ductile:



We will focus our attention on globally brittle fracture of structures.

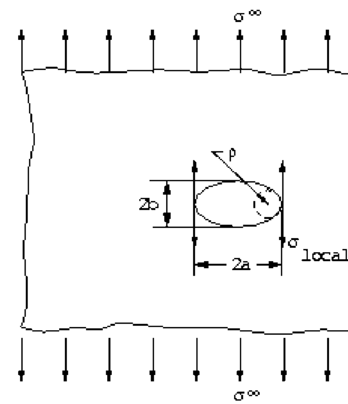
A Fracture Criterion



a_0 interatomic spacing, $\sigma_{\text{local}} >> \sigma^{\infty}$

$$\boxed{\sigma_{\text{local}} \leq \sigma_c, \quad \sigma_c - \text{ideal cleavage strength}}$$

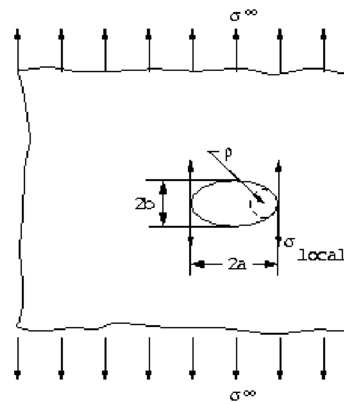
Estimate for σ_{local}



$$\sigma_{\text{local}} = (1 + 2\frac{a}{b}) \times \sigma^{\infty}, \quad \rho = b^2/a, \quad b = \sqrt{\rho\, a}$$

$$\sigma_{\text{local}} = K_t\, \sigma^{\infty}, \quad K_t = \left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

K_t is called the **theoretical stress concentration factor**, it is a **dimensionless** quantity.



$$\sigma_{\text{local}} = K_t\,\sigma^{\infty}, \quad K_t = \left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

Sharpest physical crack, $\rho_{\text{min}} \simeq a_0$:

$$\boxed{\sigma_{\text{local}} = K_{t,max}\,\sigma^{\infty} \quad K_{t,max} = 1 + 2\sqrt{a/a_0} \doteq 2\sqrt{\frac{a}{a_0}}, \quad a/a_0 \gg 1}$$

$a_0 \approx 0.3 \times 10^{-9}$ m, $2a = 10^{-6}$ m, $a \doteq 1650\ a_0$

$\Rightarrow \quad K_t = 1 + 2\sqrt{1650} = 1 + 81.2 \doteq 81.2$!

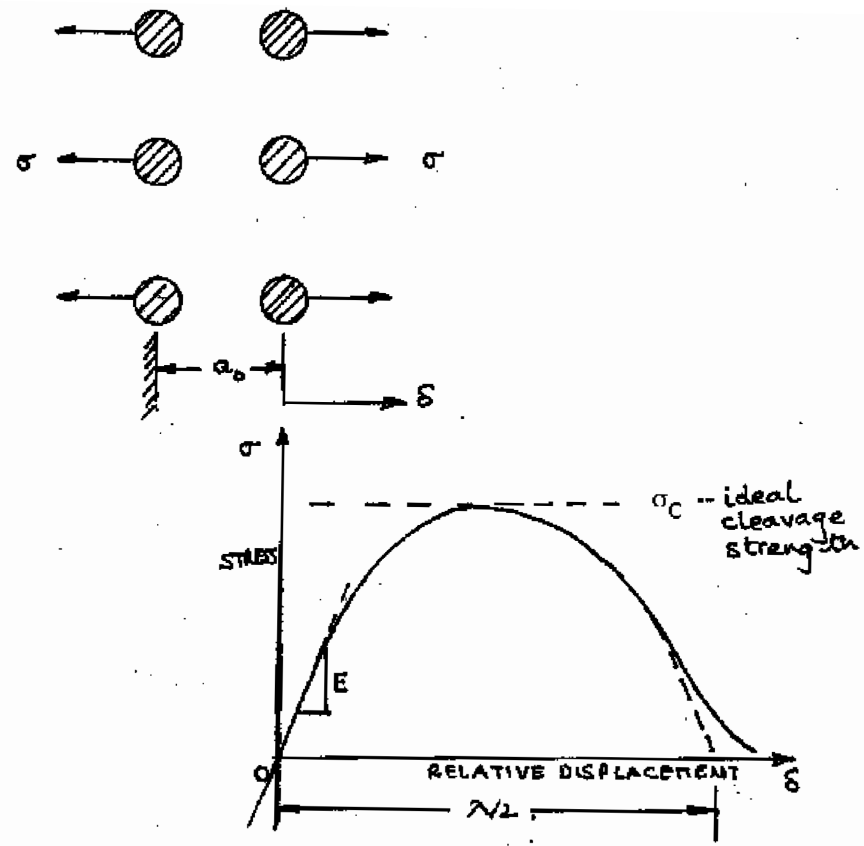
Estimate for the ideal cleavage strength σ_c

- A perfect single crystal is expected to be the strongest form of a crystalline solid, so its strength represents an upper bound to the attainable strength of a solid in the absence of any cracks. On the atomic scale, fracture is always a tensile phenomenon in which planes of atoms are pulled apart.

- The "ideal cleavage strenth"

$$\sigma_c,$$

represents the stress required to separate two neighboring planes of the atomic lattice. This ideal cleavage strength is associated with a theoretically perfect crystal, and it is determined solely by the binding forces between the atoms.



$$\sigma(\delta) \doteq \begin{cases} \sigma_C \sin(2\pi\delta/\lambda) & \text{for } 0 \le \delta \le \lambda/2, \\ 0 & \text{for } \delta > \lambda/2. \end{cases}$$

$$\sigma(\delta) \doteq \sigma_c \sin(2\pi\delta/\lambda) \quad \text{for } 0 \le \delta \le \lambda/2$$

$$\sigma(\delta) \doteq \sigma_c \, (2\pi\delta/\lambda) \quad \text{for } \delta << \lambda/2$$

$$\sigma = E\epsilon = E \times \frac{\delta}{a_0} \quad \text{for } \delta << \lambda/2$$

$$\Rightarrow$$

$$\sigma_c \times \frac{2\pi\delta}{\lambda} = E \times \frac{\delta}{a_0} \quad \text{for } \delta << \lambda/2$$

$$\Rightarrow$$

$$\sigma_c = (\frac{E}{\pi}) \times (\frac{\lambda/2}{a_0})$$

The length $(\lambda/2)$ was introduced to represent the effective range of the atomic forces. We estimate $(\lambda/2)$ by applying an energy balance to the idealized separation process.

The external work per unit area required to separate a crystal across a lattice plane is simply the area under the σ vs. δ curve. The fracture process creates two new surfaces in the crystal, thus the total energy required to produce the two surface is $2\Gamma_s$, where Γ_s is the **surface energy per unit area** of the crystallographic cleavage planes.

$$\int_{\delta=0}^{\delta=\infty} \sigma(\delta)\, d\delta = 2\Gamma_s,$$

$$\int_{\delta=0}^{\delta=\infty} \sigma(\delta)\, d\delta \doteq \int_{\delta=0}^{\delta=\lambda/2} \sigma_c \sin(\frac{2\pi\delta}{\lambda})\, d\delta = \sigma_c \times (\frac{\lambda}{\pi}),$$

$$\Rightarrow \quad \sigma_c \times (\frac{\lambda}{\pi}) = 2\Gamma_s,$$

$$\Rightarrow \quad \frac{\lambda}{2} = \frac{\pi\,\Gamma_s}{\sigma_c}.$$

$$\sigma_c = (\frac{E}{\pi}) \times (\frac{\lambda/2}{a_0}), \qquad \frac{\lambda}{2} = \frac{\pi\,\Gamma_s}{\sigma_c}.$$

$$\Downarrow$$

$$\boxed{\sigma_c = \sqrt{\frac{E\,\Gamma_s}{a_0}}}$$

Thus, when looking for strong crystalline solids, we should look for materials having high elastic modulus and specific surface energy, and with compact crystallographic unit cells.

The specific surface energy Γ_s of most solids scales with the elastic modulus E, and when expressed in units of $(E\,a_0)$ J/m^2, is given to an adequate approximation by

$$\Gamma_s \simeq \frac{E\,a_0}{100} \; - \; \frac{E\,a_0}{10}.$$

Substituting this in $\sigma_c = \sqrt{\frac{E\,\Gamma_s}{a_0}}$ we obtain

$$\boxed{\sigma_c \simeq \frac{E}{10} - \frac{E}{3}.}$$

This represents an upper bound to the attainable strength in a perfect crystalline solid in the absence of cracks.

However, most solids contain intrinsic crack-like micro-defects, and for this reason they generally fracture at stress levels one to three orders of magnitude smaller than this estimate for σ_c.

Recall

$$\sigma_{\text{local}} = K_{t,max}\,\sigma^{\infty},\;\; K_{t,max} \doteq 2\,\sqrt{\frac{a}{a_0}},\;\; \Rightarrow\;\; \sigma_{\text{local}} = \sigma^{\infty} \times 2\sqrt{\frac{a}{a_o}}$$

$$\sigma_{\text{local}} \leq \sigma_c$$

$$\sigma^{\infty} \times 2\sqrt{\frac{a}{a_o}} \leq \sigma_c$$

$$\sigma^{\infty}\sqrt{\pi a} \leq \left\{\frac{\sqrt{\pi\, a_o}}{2}\,\sigma_c\right\}$$

$$\boxed{K_I \leq K_{Ic}, \quad \text{with} \quad K_I = \sigma^{\infty}\sqrt{\pi\, a}, \quad \text{and} \quad K_{Ic} = \left\{\frac{\sqrt{\pi\, a_o}}{2}\,\sigma_c\right\}}$$

The quantity K_I is called the mode I **stress intensity factor**. The quantity K_{Ic} is a **critical stress intensity factor**, it is a **material property** also called the **fracture toughness**.

$$K_{Ic} = \left\{ \frac{\sqrt{\pi\, a_o}}{2}\, \sigma_c \right\}, \quad \sigma_c \simeq \frac{E}{5}, \quad a_o \simeq 0.3 \times 10^{-9}\text{m}$$

$$\Rightarrow \quad K_{Ic}\big|_{\text{min}} \simeq \left\{ 3 \times 10^{-6} \times E \right\} \text{MPa}\sqrt{\text{m}}.$$

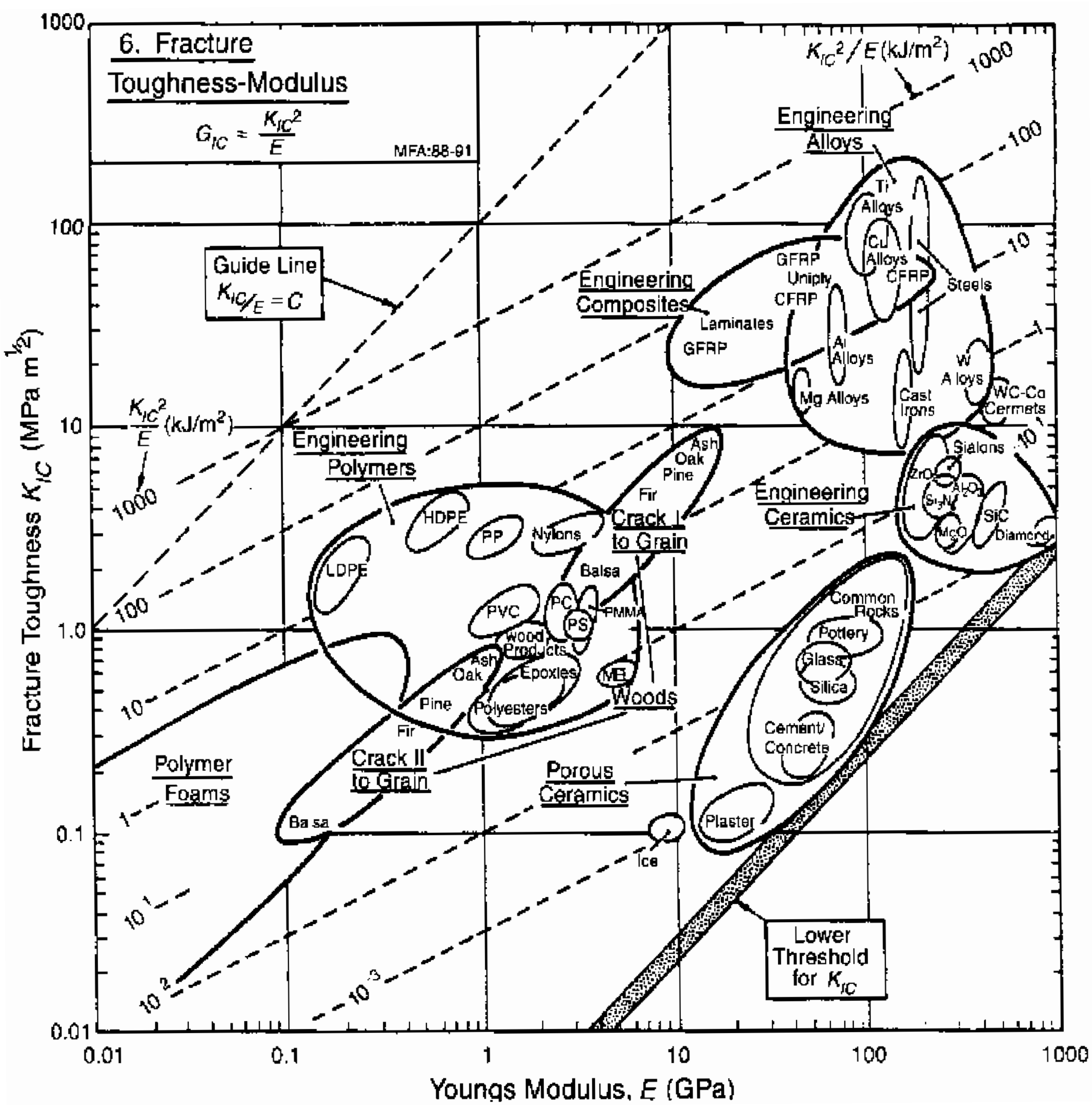
- This estimate defines a lower limit on values of K_{Ic}, since in obtaining this estimate we have neglected the effects of plasticity within the process zone.

- It is reasonably accurate for most brittle ceramics and glasses because when they fracture, there is negligible inelastic deformation, and the energy absorbed is only slightly more than the surface energy.

- In contrast, when metals and ductile polymers fracture, the energy absorbed is vastly greater because of the plasticity associated with crack initiation and propagation. The effect of local plasticity in the fracture process zone is to raise the values of K_{Ic} to values much larger than the purely elastic estimate given by

$$K_{Ic}|_{\mathrm{min}} \simeq \left\{ 3 \times 10^{-6} \times E \right\} \ \mathrm{MPa}\sqrt{\mathrm{m}}.$$

- **In practice, the material property K_{Ic} is determined experimentally from combinations of crack size and applied stress at fracture.**



6. Fracture
Toughness-Modulus
$G_{IC} \approx \frac{K_{IC}^2}{E}$
MFA:88-91
Guide Line
$K_{IC}/E = C$
K_{IC}^2/E (kJ/m²)
Engineering Alloys
Ti Alloys
Cu Alloys
CFRP
Steels
GFRP Uniply
CFRP
Laminates
GFRP
Engineering Composites
Al Alloys
Mg Alloys
Cast Irons
W Alloys
WC-Co Cermets
Sialons
ZrO_2
Al_2O_3
Si_3N_4
SiC
MgO
Diamond
Engineering Ceramics
Engineering Polymers
HDPE
PP
Nylons
LDPE
PVC
PC
PS
PMMA
Balsa
Wood Products
Epoxies
MEL
Polyesters
Ash Oak Pine Fir
Crack ⊥ to Grain
Crack ∥ to Grain
Woods
Polymer Foams
Balsa
Porous Ceramics
Plaster
Ice
Common Rocks
Pottery
Glass
Silica
Cement/ Concrete
Lower Threshold for K_{IC}
Fracture Toughness K_{IC} (MPa m^1/2)
Youngs Modulus, E (GPa)

SUMMARY

The macroscopic fracture criterion for globally brittle fracture of structures is

$$\boxed{K_I \leq K_{Ic}, \qquad K_I = \sigma^{\infty}\sqrt{\pi\, a}, \qquad K_{Ic} - \text{Fracture Toughness.}}$$

The quantity K_1 is called the mode I **stress intensity factor**. It is a function of the applied stress σ^{∞} and the crack length a, it has units of MPa$\sqrt{\text{m}}$.

The quantity K_{Ic} is a **critical stress intensity factor**. It a material property which is also called the **fracture toughness**. It measures the resistance of a material to the propagation of a crack.

Typical applications of $\sigma^{\infty}\sqrt{\pi\, a} \leq K_{Ic}$ include:

1. **Given**: Fracture toughness K_{Ic} and Crack size a.
   Determine: Critical stress at fracture,
$$\sigma_f = \frac{K_{Ic}}{\sqrt{\pi a}}.$$

2. **Given**: Fracture toughness K_{Ic} and applied stress σ^{∞}.
   Determine: Critical crack size at fracture,
$$a_c = \frac{1}{\pi}(\frac{K_{Ic}}{\sigma^{\infty}})^2.$$

3. **Given**: Stress and crack size at fracture, σ_f and a_f.
   Determine: Fracture toughness, K_{Ic},
$$K_{Ic} = \sigma_f \sqrt{\pi a_f}.$$

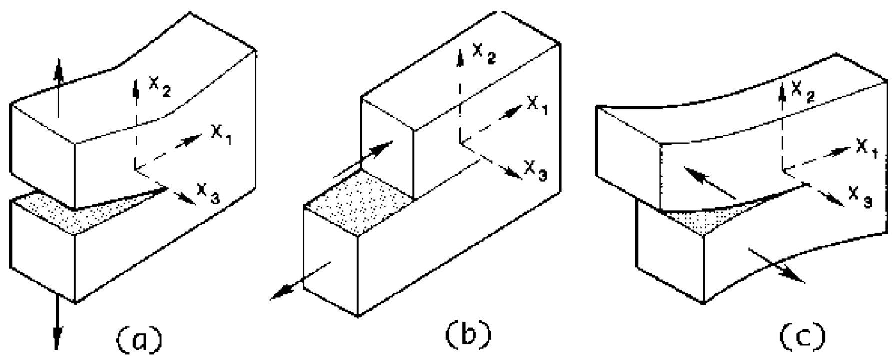
In the what follows, we will

- Examine the foundations of linear elastic fracture mechanics more deeply, noting special results in linear elastic crack tip stress analysis

- Take consideration of the size and shape of the crack tip plastic zones, and

- Consider the inherent limits of applicability of linear elastic fracture mechanics.

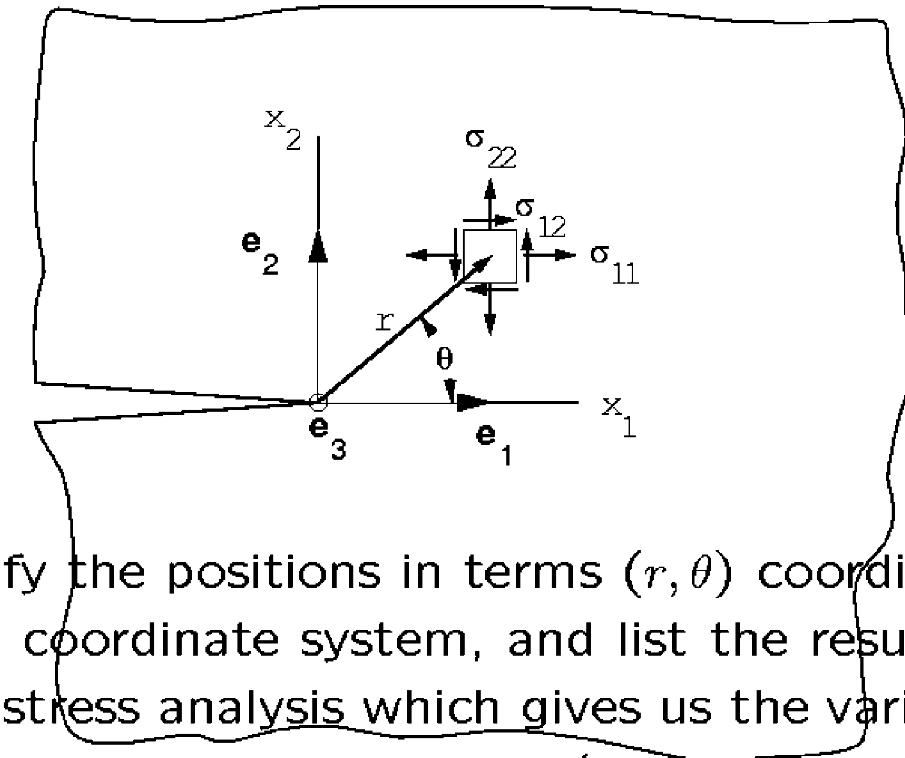
Consider a sharp crack in a prismatic isotropic linear elastic body. There are three basic loading modes associated with relative crack face displacements for a cracked body.



(a) the tensile opening mode, or Mode I
(b) the in-plane sliding mode, or Mode II
(c) the anti-plane tearing mode, or Mode III.

In most engineering applications, brittle fracture of structures typically occurs under Mode I. Accordingly, in what follows, we shall focus on Mode I only.

Stress field in the vicinity of the crack-tip:



We shall identify the positions in terms (r, θ) coordinates of a cylindrical coordinate system, and list the results of an asymptotic stress analysis which gives us the variation of the components σ_{ij} with position (r, θ) as $r \to 0$, for Modes I.

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \begin{pmatrix} 1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) \\ 1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) \end{pmatrix} + \text{bounded terms}$$

$$\sigma_{33} = \nu\left(\sigma_{11} + \sigma_{22}\right), \quad \sigma_{13} = \sigma_{23} = 0, \qquad \text{for plane strain}$$

$$\sigma_{33} = \sigma_{13} = \sigma_{23} = 0, \qquad \text{for plane stress}$$

The stress components are proportional to $1/\sqrt{r}$, and they approach infinity as $r \to 0$. **Mathematical singularity**.

The stress components are proportional to the parameter K_I. It is for this reason that K_I is called the **the Mode I stress intensity factor**.

- The task of stress analysis in linear elastic fracture mechanics (LEFM) is to evaluate K_I and its dependence on geometry, load type, and load amplitude.

- Over the last ∼40 years many methods for obtaining K-solutions have been developed. Several hundred solutions, mostly for two-dimensional configurations, are now available.

- The common methods for finding the appropriate K-solutions are listed below:
  - Look up K for the geometry of interest in a Handbook of Stress Intensity Factors.
  - Perform a numerical stress analysis (e.g., finite element) of the problem, and use an appropriate method of post-processing the results to extract the stress intensity factor.
  - Experimental stress analysis of the configuration. Among the experimental tools available are photoelasticity, Moiré interferometry, and careful measurement of the overall structural compliance (Generalized [Displacement] / [Force] ratio) as a function of crack dimensions.

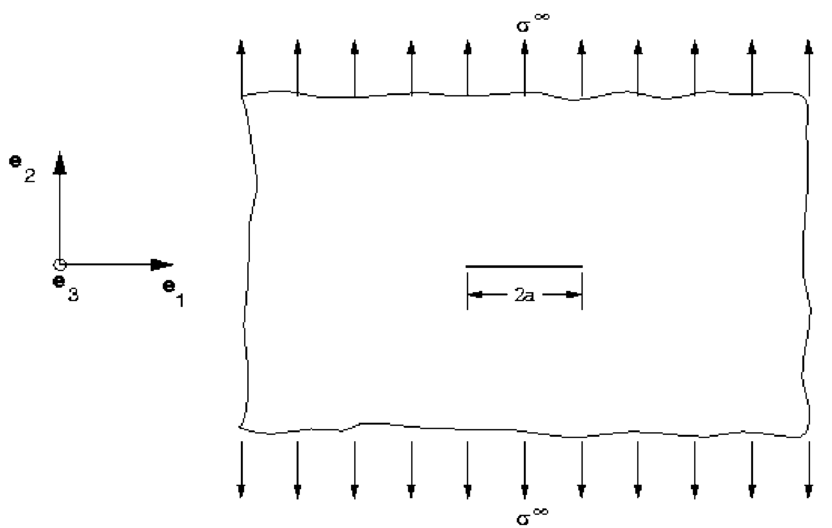
For simple geometries, or where a complex structure can be simply modeled, it may be possible to use the following handbooks:

1. Tada, H., Paris, P. C., and Irwin, G. R., *The Stress Analysis of Cracks Handbook. Second Edition*. Paris Productions Incorporated, 226 Woodbourne Dr., St. Louis, Missouri 63105, 1985.

2. Rooke, D. P. and Cartwright, D. J., *Compendium of Stress Intensity Factors*, HMSO, London, 1976.

3. Sih, G. C., *Handbook of Stress Intensity Factors for Researchers and Engineers, Lehigh University Report*, 1976.

For the important proto-typical case of a finite crack of length $2a$ in a large body subject to a far-field stress $\sigma_{22}^{\infty} \equiv \sigma^{\infty}$,



the mode I stress intensity factor is given by

$$\boxed{K_I = \sigma^{\infty}\sqrt{\pi a}}$$

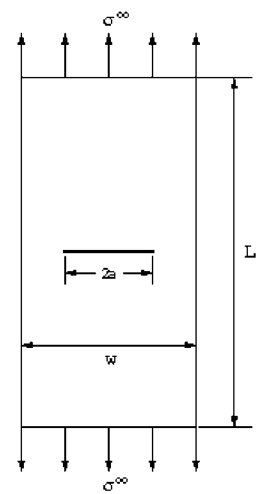
For other geometrical configurations, in which a characteristic crack dimension is a and a characteristic applied tensile stress is σ^∞, we will write the corresponding stress intensity factor as

$$\boxed{K_I = Q\,\sigma^\infty\sqrt{\pi a}}$$

where Q is a dimensionless factor needed to account for a geometry different from that of our proto-typical case.

We call Q the **configuration correction factor**. It is usually given in terms of dimensionless ratios of relevant geometrical quantities.

For example, for a center crack in a long ($L > 3w$) strip of finite width w,



the stress intensity factor is given by

$$K_I = Q\sigma^{\infty}\sqrt{\pi a}, \qquad Q = \hat{Q}\left(\frac{a}{w}\right) \doteq \left\{\sec\left(\frac{\pi a}{w}\right)\right\}^{1/2}.$$

It is important to note that the stress intensity factors for combined loading can be obtained by superposition.

That is, if for a given cracked body the far-field loading can be decomposed into

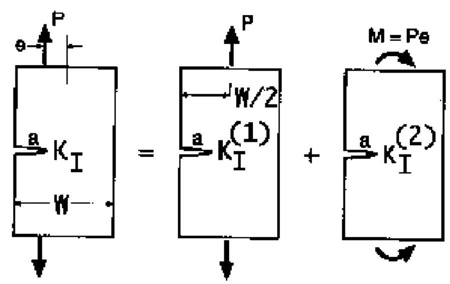
$$\sigma^{\infty} = \sigma^{\infty\,(1)} + \sigma^{\infty\,(2)},$$

then K_I for the cracked configuration under a far-field stress σ^{∞} can be obtained as

$$K_I = K_I^{(1)} + K_I^{(2)},$$

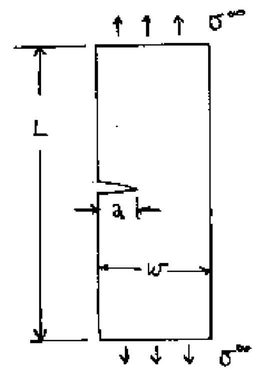
where $K_I^{(1)}$ and $K_I^{(2)}$ correspond to the stress intensity factors for the cracked configuration under the far-field stresses $\sigma^{\infty\,(1)}$ and $\sigma^{\infty\,(2)}$, respectively.

As an example, consider an eccentric load P applied at a distance e from the centerline of a component with an edge crack:



This eccentric load is statically equivalent to the combination of a centrally applied load P and a bending moment $M = Pe$. Then, using the stress intensity factors for (1) an edge crack in a finite width strip in tension, and (2) an edge crack in a finite width strip subjected to pure bending, the stress intensity factor for the eccentrically loaded strip with an edge crack can be easily calculated.

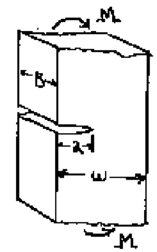
Edge Crack in a Long ($L > 3w$) Finite Width Strip in Tension



$$K_I = Q\sigma^{\infty}\sqrt{\pi a}$$

$$Q = \hat{Q}\left(\frac{a}{w}\right) \doteq \frac{1.12}{\left[1 - 0.7\left(\frac{a}{w}\right)^{1.5}\right]^{3.25}} \;,\; \frac{a}{w} \leq 0.65.$$

Edge Crack in a Long ($L > 3w$) Finite Width Strip of Thickness B Subject to a Pure Bending Moment M



$$K_I = Q\,\sigma_{\text{nom}}\sqrt{\pi a} \qquad \sigma_{\text{nom}} = \frac{6M}{Bw^2}.$$

$$Q \doteq \frac{1.12}{\left[1 - \left(\frac{a}{w}\right)^{1.82}\right]^{1.285}} - \sin\left(\frac{\pi}{2}\frac{a}{w}\right);\ \ \frac{a}{w} \leq 0.7$$

Deep Crack Approximation for $a/w > 0.7$:

$$K_I \doteq (4M)/Bc^{3/2}, \quad \text{where} \quad c \equiv w - a.$$

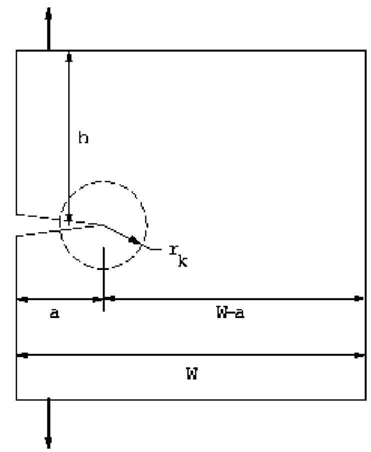
LIMITS OF APPLICABILITY OF K_I-SOLUTIONS

1. In order for the asymptotic field

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \quad \text{as} \quad r \to 0,$$

based on K_I to acceptably approximate the complete elastic fields at a finite distance r from a crack tip, it is necessary that r be no more distant from the crack tip than a few percent of other characteristic in–plane dimensions, such as the crack length, the remaining uncracked ligament, and distance from the crack tip to a point of application of load.

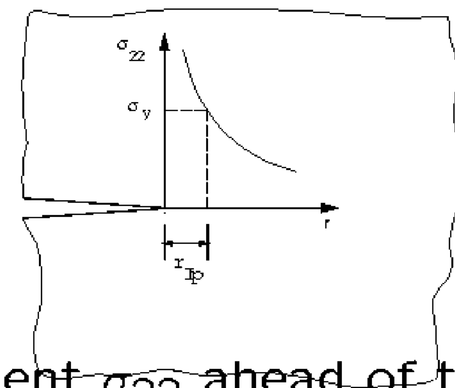
With respect to the figure below,



the asymptotic fields based on K_I should acceptably approximate the complete elastic fields at distances up to r_K from the crack-tip, provided $r_K \ll a$, $(W-a)$, h.

K_I dominant region : $\{r \mid r < r_K \; ; \;\; r_K \ll a, \, (W-a), \, h, \}$

2. **Limit to applicability of K_I-solutions because of the local inelastic deformation**



The stress component σ_{22} ahead of the crack is

$$\sigma_{22}(r, \theta = 0) = \frac{K_I}{\sqrt{2\pi r}} .$$

This exceeds σ_y at points closer than the distance

$$r_{Ip} \doteq \frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2 .$$

The length r_{Ip} is called the plastic zone size.

Small Scale Yielding (s.s.y)

For Mode I loading, yielding at the tip of a crack will occur over a region whose maximum dimension is given by

$$r_{Ip} \doteq \frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2 .$$

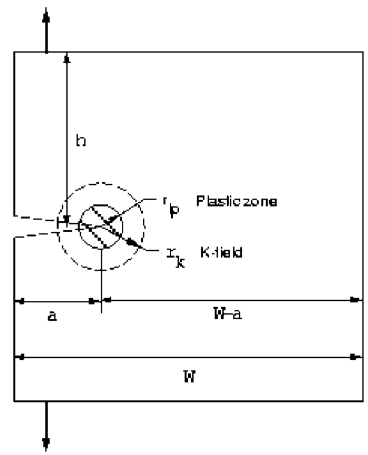
Linear elastic fracture mechanics is based on the concept of small scale yielding (s.s.y.).

Small scale yielding is said to hold when the applied loads levels are sufficiently small (less than approximately one-half of the general yielding loads) such that there exists a radius $r = r_K$ about the crack-tip (exterior to the plastic zone) with the following properties:

- $r_{Ip} \ll r_K$, so that the stress fields at r_K is free of any perturbations due to plasticity.

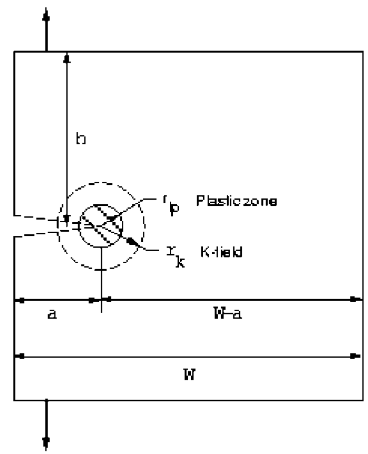
- $r_K \ll$ {crack length, relevant in-plane geometric dimensions} , so that the elastic solution at r_K is given accurately in terms of the K_1-solution.

From a practical viewpoint, it is necessary that $r_{Ip} << a, (W-a)$, and h:



A distance of $15 \times r_{Ip}$ is generally considered to be sufficient for small scale yielding conditions to prevail. Hence,

$$\text{if} \quad a,\ (W-a),\ h \gtrsim 15 \times \frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2, \quad \text{then s.s.y. holds,}$$



Under small scale yielding conditions the asymptotically computed stress field

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \quad \text{as} \quad r \to 0,$$

is close to the "complete" stress field for all material points on $r = r_K$. The stress magnitude for all material points on $r = r_K$ is governed solely by the value of K_I. **That is, there is a one-parameter, K_I, characterization of the crack tip region stress field.**

Since the value of the stress field for all material points on r_K is governed solely by the value of K_I, the specific physical processes of material separation occurring at the crack tip are driven by K_I.

In the theory of linear elastic fracture mechanics (LEFM) the criterion for fracture initiation or crack extension in response to slowly applied loading of monotonically increasing magnitude is taken as

$$\boxed{K_I \leq K_c,}$$

where K_I is the applied **stress intensity factor**, and K_c is a material and thickness dependent critical value of K_I for crack extension, called the **fracture toughness**.

The fracture toughness for a given material and component thickness is determined experimentally.

FRACTURE TOUGHNESS TESTING

Fracture toughness values are obtained by testing fatigue-cracked specimens (to prepare cracks of relatively standard sharpness) at a given temperature and loading rate.

Standard specimen geometries and test procedures used to obtain such data at slow loading rates are given in American Society for Testing and Materials (ASTM) Standard E-399.

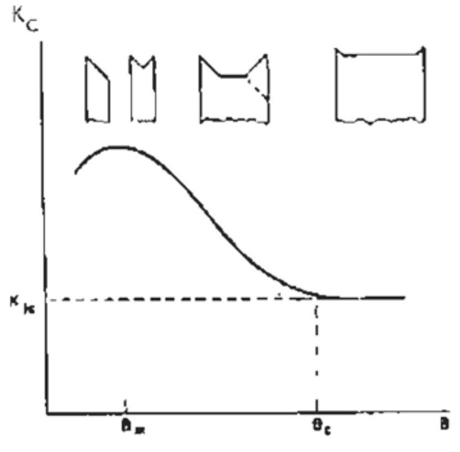
The test methods have been fully discussed in the laboratory.

For operational purposes the E-399 standard requires

$$a,\ (W-a),\ h \gtrsim 15 \times r_C, \qquad r_C \equiv \frac{1}{2\pi}\left(\frac{K_c}{\sigma_y}\right)^2$$

If this requirement of small scale yielding is met, then a **valid** K_c value has been obtained in the test.

For a given material the fracture toughness value K_c generally depends on the thickness B of the test specimen.



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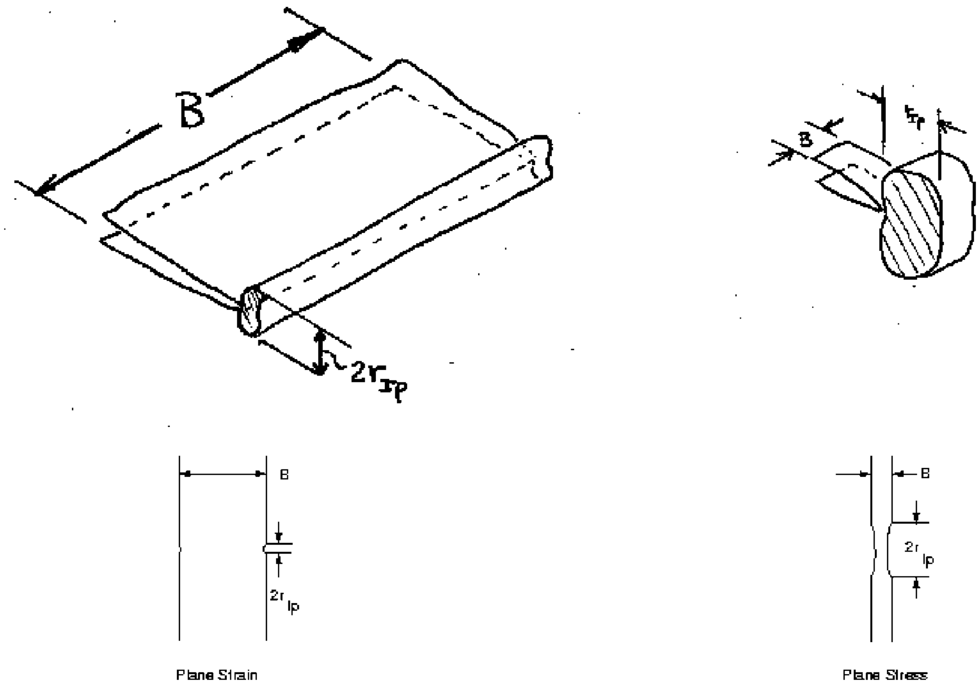
- Note that regardless of the value of B, we expect that an unconstrained planar elastic body loaded in its plane will globally be in a state of plane stress.

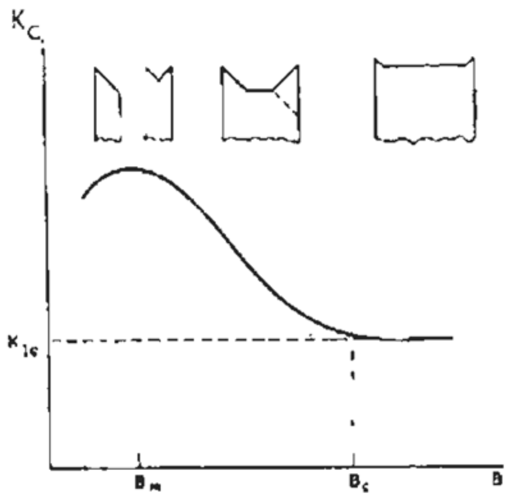
- However, when plastic deformation occurs at the crack-tip, the notions of "plane-stress" or "plane-strain" in the plastic zone at the crack tip are governed by the size of the plastic zone relative to the specimen thickness B.

- In plane-strain it is essential that $r_{Ip} \ll B$

- In plane stress $r_{Ip} \gtrsim B$.



If $r_{Ip} \ll B$, then along a very large part of the crack tip **the plastic strain field will be essentially one of plane strain**. On the other hand, if $r_{Ip} \gtrsim B$, **the plastic strain field will be essentially one of plane stress**.



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Since the plastic strain field in the vicinity of the crack tip for large values of the thickness B corresponds essentially to plane plastic strain,

$$K_{Ic} \quad \equiv \quad \textbf{plane strain fracture toughness}.$$

Further, since K_{Ic} is **thickness independent** it is a **material property**.

The quantity

$$\boxed{r_{Ic} \equiv \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2}$$

is called the **plane strain critical crack tip plastic zone size**.

Thus, the second requirement of the ASTM E399 plane strain fracture toughness test is that the thickness B satisfy

$$\boxed{B \gtrsim 15 \times \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2 .}$$

If a test is performed on specimens which satisfy the geometric requirement on the in-plane dimensions for small scale yielding,

$$\boxed{a,\,(W-a),\,h \gtrsim 15 \times \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2,}$$

and also the requirement on the thickness

$$\boxed{B \gtrsim 15 \times \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2.}$$

for plastic plane strain at the crack tip, then the measured fracture toughness will be a **valid plane strain fracture toughness**, K_{Ic}.

	E, GPa	σ_y, MPa	K_{Ic}, MPa$\sqrt{\text{m}}$
Steels			
AISI-1045	210	269	50
AISI-1144	210	540	66
ASTM A470-8	210	620	60
ASTM A533-B	210	483	153
ASTM A517-F	210	760	187
AISI-4130	210	1,090	110
AISI-4340	210	1,593	75
200-Grade Maraging	210	1310	123
250-Grade	210	1,786	74
Aluminum Alloys			
2014-T651	72	415	24
2024-T4	72	330	34
2219-T37	72	315	41
6061-T651	72	275	34
7075-T651	72	503	27
7039-T651	72	338	32
Titanium Alloys			
Ti-6AL-4V	108	1,020	50
Ti-4Al-4Mo-2Sn-0.5Si	108	945	72
Ti-6Al-2Sn-4Zr-6Mo	108	1,150	23

	E, GPa	K_{Ic}, MPa$\sqrt{\text{m}}$
Polymers		
Epoxies	3	0.3 − 0.5
PS	3.25	0.6 − 2.3
PMMA	3. − 4.	1.2 − 1.7
PC	2.35	2.5 − 3.8
PVC	2.5 − 3.	1.9 − 2.5
PETP	3	3.8 − 6.1
Ceramics		
Soda-Lime Glass	73	0.7
MgO	250	3
Al_2O_3	350	3. − 5.
Al_2O_3, 15% ZrO_2	350	10.
Si_3N_4	310	4. − 5.
SiC	410	3.4

- The K_{Ic} value of many structural materials under a constant slow rate of loading increases with increasing temperature.

Image removed due to copyright considerations. Graph of fracture toughness versus temperature.

– For most structural steels the rate of increase of K_{Ic} with temperature is not constant, but increases markedly above a certain test temperature called the "transition temperature".

– Below the transition-temperature, the mode of crack extension is predominantly cleavage requiring little work to fracture

– Above the transition temperature the crack extension is by ductile tearing, requiring large amounts of work to fracture.

– In the transition temperature region there is a continuous change in the microscopic mode of crack extension.

- The plane strain fracture toughness value, K_{Ic}, is also affected by the loading rate.

  For structural steels it is found that as the loading rate increases, the value of K_{Ic} decreases.

  The asymptotic value of K_{Ic} under impact or "dynamic" loading rates is denoted by K_{Id}.

  It is found that variation of K_{Id} with temperature is similar to that of K_{Ic}.

- The K_{Ic} value determined at a temperature and loading rate corresponding to service conditions is used to estimate the relation between fracture stress and crack size for a material in service when small-scale-yielding and plane strain conditions are expected.

- Even when plane strain crack tip conditions are not expected, the crack extension criterion $K_I \leq K_{Ic}$ is sometimes adopted instead of the more appropriate criterion $K_I \leq K_c$, because difficulties exist in establishing K_c values for various plate thicknesses. As is clear, this will lead to conservative design. Alternative design procedures exist, and should be explored.

- Further, cyclic loads can cause crack extension at maximum K_I values less than the K_{Ic} value. Crack extension under cyclic or sustained load will be increased by the presence of an aggressive environment. Therefore, application of K_{Ic} in the design of service components must take such cyclic and environmental crack extension into account by referring to appropriate information.

A TYPICAL PROCEDURE FOR DESIGN TO PREVENT BRITTLE FRACTURE OF COMPONENTS

1. Determine the location, magnitude and orientation of the maximum principal stress $\sigma \equiv \sigma_I$ in the component.

   Recall that for an arbitrary stress state we can always find the principal stresses $\{\sigma_I \geq \sigma_{II} \geq \sigma_{III}\}$ and the corresponding principal directions $\{\mathbf{n}_I, \mathbf{n}_{II}, \mathbf{n}_{III}\}$. It is emphasized that the subscripts I, II, and III used here **do not** refer to the three different modes of loading at a crack front.

2. Determine the location, size and orientation of any cracks in the component by non-destructive testing (NDT) techniques.

   If no cracks are detected, then assume that the crack size a **that can escape detection by the NDT technique is present, and is in the worst possible location and orientation.**

3. Determine the stress intensity factor $K_I = Q\,\sigma\sqrt{\pi a}$ for these conditions of stress σ, crack length a, and geometry of the cracked body.

The first place to search for the appropriate expressions for K_I are the handbooks tabulating stress intensity factors. If no tabulated case is found, then appropriate numerical and/or experimental techniques need to be employed to determine the expression for $K_I = Q\,\sigma\sqrt{\pi a}$.

4. Determine K_{Ic} for the material from which the component is built.

   Values for K_{Ic} have been tabulated for some important engineering materials. Be careful, K_{Ic} values are in general a function of the temperature and the loading rate. Use the appropriate value of K_{Ic} for the service conditions. If an appropriate value for K_{Ic} cannot be found, then suitable experiments will need to conducted to determine the plane strain fracture toughness for the material under the service conditions.

5. Use suitable factors of safety, S, in your design based on the governing equation $K_I = (K_{Ic}/S)$.

6. Ensure that $K_I < (K_{Ic}/S)$ throughout the entire life of the structure.

   Be careful of fatigue due to cyclic loading. A component which might have been properly designed against brittle fracture under static loading, might fail under cyclic loading if the crack grows to point when the critical condition $K_I = K_{Ic}$ is met.

   Failure due to fatigue will be our next topic of study.