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SUPPLEMENTARY NOTES

FATIGUE CRACK PROPAGATION

We have emphasized that most engineering materials contain small crack-like defects, or they can readily develop them during service.

If a crack exists in a structure, then the stress field in the vicinity of the crack tip is given by

$$\sigma_{ij}(r,\theta) \to \frac{K_I}{\sqrt{2\pi r}}\, f_{ij}(\theta) \quad \text{as } r \to 0,$$

where the stress intensity factor is

$$K_I = Q\,\sigma\,\sqrt{\pi a}\ ,$$

with σ the far-field applied Mode I tensile stress, a the crack length, and Q the configuration correction factor.

Under small–scale yielding, and when *monotonically increasing* far-field load is applied to the body, then a necessary condition for crack extension is

$$K_I = K_{Ic},$$

where K_{Ic} is a material property called the plane strain fracture toughness.

The above criterion for fracture is deceptively simple. In practice there are problems associated with identifying the shape and size of a crack, with carrying out a proper stress analysis, and in obtaining accurate and/or valid data for K_{Ic}. Also, *cracks can extend in a sub-critical manner,* which means that a component initially thought to be safe against fracture may become dangerous after a period of service. Subcritical crack nucleation and growth can occur under *constant* or *fluctuating* loads. In the former case, crack extension is usually controlled by an aggressive environment which causes *stress corrosion cracking*; we shall return to a study of this phenomenon later. Subcritical crack nucleation and growth under fluctuating loads is called "fatigue." It is this phenomenon to which we now turn our attention.

Failure occurring from repeated fluctuating stresses or strains is called fatigue. The word "fatigue" was introduced in the 1840's and 1850's in connection with such failures which occurred in the then rapidly developing railway industry. It was found that railroad axles failed regularly at shoulders, and that these failures appeared to be quite different from failures associated with monotonic testing. Even then, elimination of sharp corners was recommended.

The *process* of fatigue failure may be defined as a process in which there is *progressive, localized, permanent microstructural change* occurring in a structure when it is subjected to boundary conditions which produce fluctuating stresses and strains at some material point or points. These microstructural changes may culminate in the formation of *cracks*

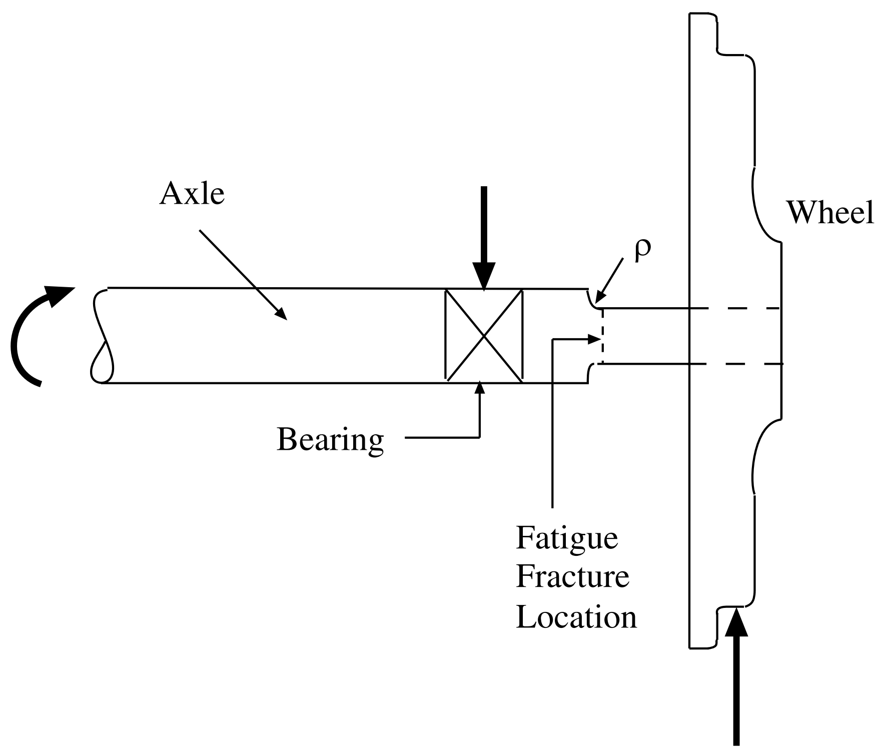

Figure 1: Schematic of rotating bending fatigue failure in railway axles.

and their subsequent growth to a size which causes final *fracture* after a sufficient number of stress or strain fluctuations.

The word *progressive* implies that the fatigue process occurs over a period of time or usage. A fatigue failure is often very sudden with no external warning; however, the mechanisms involved may have been operating since the beginning of the time when the component or structure was put to use.

The word *localized* implies that the fatigue process operates at local areas rather than throughout the body. These local areas can have high strains and stresses due to abrupt changes in geometry and material imperfections.

The phrase *permanent microstructural changes* emphasizes the central role of cyclic plastic deformations in causing irreversible changes in the substructure. Countless investigations have established that *fatigue results from cyclic plastic deformation in every instance,* even though the structure as a whole is practically elastic. A small plastic strain excursion applied only once does not cause substantial changes in the microstructure of ductile materials, but multiple repetitions of very small plastic deformations leads to cumulative damage ending in fatigue failure. We note that although fatigue is popularly associated with metallic materials, it can occur in all engineering materials capable of undergoing plastic deformation. This includes polymers, and composite materials with plastically deformable phases. Plastically non-deformable materials such as glasses and ceramics, in which deformation is truly elastic everywhere, do not fail by fatigue due to repeated stresses. However, recent data has shown that ceramics can exhibit fatigue crack growth under certain circumstances. This process is still consistent with our definition in the sense that local irreversible deformation at the crack tip associated with processes such as microcracking, frictional sliding, particle detachment and crack face wedging are involved in the fatigue process. Furthermore, these local mechanisms in brittle materials can give rise to macroscopic behavior which is phenomenologically similar to plasticity.

Crack-tolerant Design and Maintenance Against Fatigue Failures

If the potential cost of a structural fatigue failure in terms of human life and dollars is very high, then the design of such engineering components and structures should be based on:

1. The assumption that all fabricated components and structures contain a pre–existing population of cracks of a minimum size. This minimum size should be taken to be the *minimum that can be* reliably *detected by non-destructive examination (NDE) methods.*
2. The requirement that none of these presumed pre-existing cracks be permitted to grow to a critical size during the expected service life of the part or structure. Normally, this requires the selection of inspection intervals within the service life.

The major aim of defect-tolerant approaches to fatigue is to predict reliably the growth of pre–existing cracks of specified initial size (a_i), shape, location and orientation in a structure subjected to prescribed cyclic loadings. Providing this goal can be achieved, then inspection and service intervals can be established such that cracks should be readily detectable well before they have grown to near critical size, a_c.

The accompanying figure schematically illustrates the overall approach. A structure is subjected to a load P which cyclically varies between maximum and minimum values, $P_{\max}$ and $P_{\min}$, respectively. This loading causes a similar cyclic variation in the remote stress level, σ. The structure is *assumed* to have a pre-existing crack of initial size a_i located at the most highly stressed location. The initial size can be either the largest pre–existing crack detected by the NDE technique used, or (if no crack was actually detected) the initial crack size is *assigned* to be a_d, where a_d is the minimum crack size which can be reliably detected by the NDE technology employed. The latter assumption is the more common.

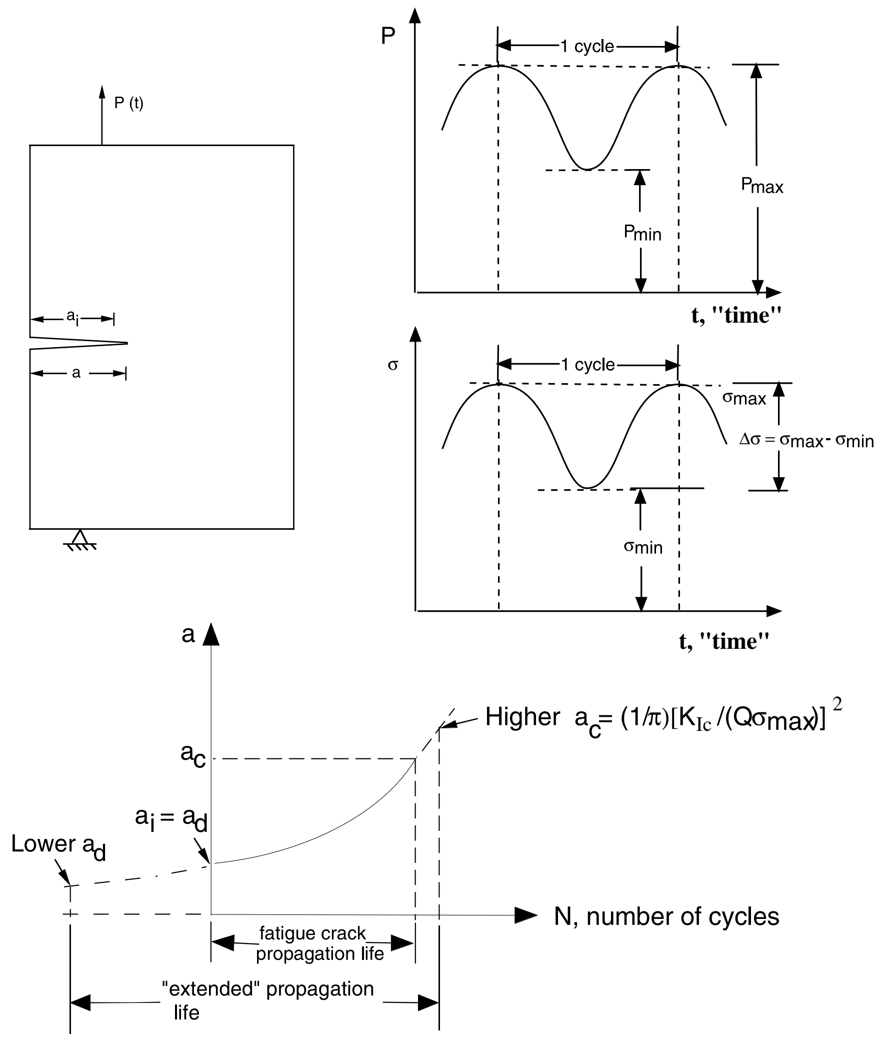

Figure 2: Crack length a versus applied cycles of loading, N

If crack growth occurs over a large number of load cycles, N, then crack length a would be observed to increase with increasing N, as shown in the figure. Failure by fracture would occur when the crack had reached the critical crack size, a_c, and the useful service life of the structure would be a suitable fraction of the number of cycles required to propagate the crack from size a_i to a_c. The critical crack size can be determined, based on reaching the plane strain fracture toughness at the maximum cyclic stress:

$$K_I = Q\,\sigma_{\max}\sqrt{\pi a_c} = K_{Ic} \Rightarrow$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Q\,\sigma_{\max}}\right)^2.$$

Also shown in the plot of a vs. N are extrapolations to values of crack length $a > a_c$ and $a < a_i \equiv a_d$. These extrapolations, of course, can be realized only by changing the initial and final conditions. If either (or both) changes could be made, it is evident that an increase in the safe service life could be realized. How could such changes be made?

First, the reduction of a_i requires a reduction in the minimum reliably-detected crack size of the NDE procedure. An increase in a_c could be obtained by raising K_{Ic} (for example, either by changing materials or by changing material processing). Although the latter option may at first seem tempting, there is far greater "pay–off" in decreasing the a_d of the NDE technique. The reason is that the growth rate (slope of the a vs. N curve) is much lower initially ($a \simeq a_i$) than at the end. A few percent increase in a_c buys only a small amount of extra service life, while a similar percentage decrease in a_d provides substantially greater increases in useful service life.

Fatigue Crack Growth

To obtain a fatigue crack growth curve for a particular application, it is necessary to establish reliable fatigue crack growth rate data. Typically, a cracked test specimen is subjected to a constant amplitude load cycling stress range $\Delta\sigma \equiv \sigma_{\max} - \sigma_{\min}$ and two curves, the crack length a versus the number of cycles, and ΔK_I for that geometry of testing versus the crack length are obtained. Here $\Delta K_I = Q\,\Delta\sigma\,\sqrt{\pi a}$ is the range of the stress intensity factor over one cycle of load, while crack length is essentially constant at value "a". Note that a considerable portion of the life of the specimen is spent at short crack lengths.

The crack growth rate is denoted by

Crack growth rate $\equiv \frac{da}{dN} \equiv$ slope of crack growth curve at crack length a.

$\equiv$ crack extension Δa of a crack of length a occurring in one cycle.

Thus, from experimentally determined curves of a vs. N and knowledge of applied loads and geometry of the test specimen we can construct $\frac{da}{dN}$ vs. a and ΔK_I vs. a curves. On cross–plotting from these curves to eliminate the variable a, we can construct $\log\left(\frac{da}{dN}\right)$ versus $\log(\Delta K_I)$ curves.

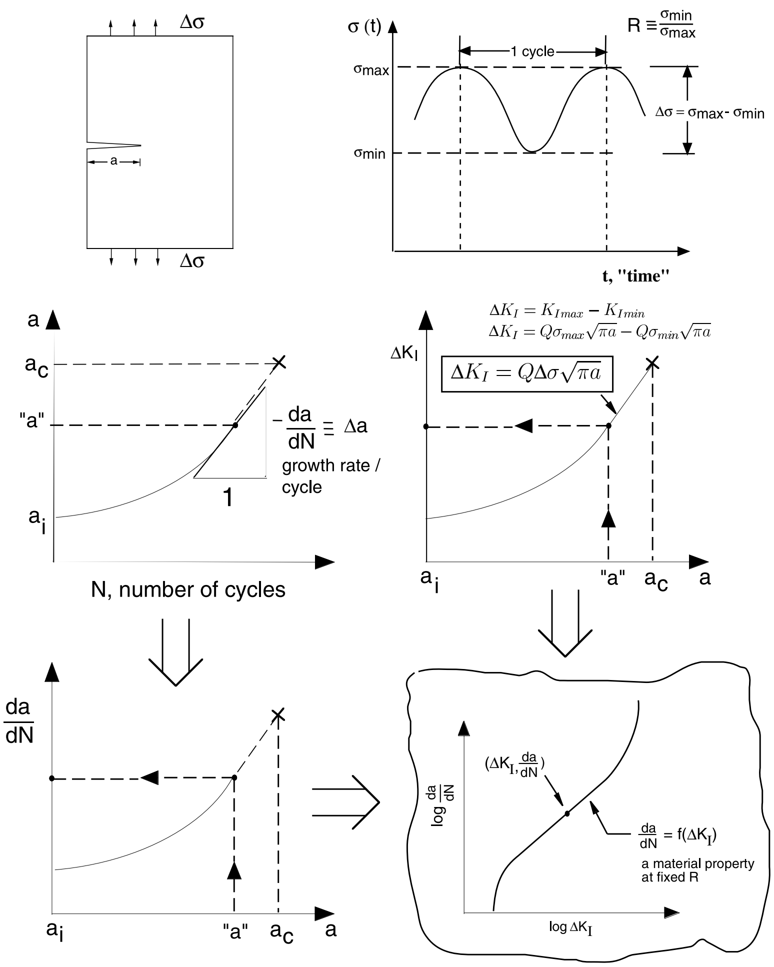

Figure 3: Schematic of fatigue crack growth-rate data reduction.

Experimentally, it has been found that for a given load ratio $R \equiv \sigma_{\min}/\sigma_{\max} = K_{I_{\min}}/K_{I_{\max}}$, the plots of $\log\left(\frac{da}{dN}\right)$ versus $\log(\Delta K_I)$ obtained from various different specimen types superpose on one another to give a single curve for a given material. The fact that such a curve can, to a good approximation, be considered to be a material curve, independent of geometrical factors, is of great practical importance: the results obtained from simple laboratory specimens can be directly applied to real service conditions, provided the stress intensity factor range in the latter case can be determined.

At a fixed R-ratio, the fatigue crack propagation behavior of metallic materials can be divided into three regimes. The boundaries dividing adjacent regimes can be specified either by the magnitude of the transitional growth rate per cycle or by the magnitude of the cyclic stress intensity factor, ΔK_I. The former distinction provides more physical insight, especially when the growth rate per cycle is compared with other relevant metallurgical length scales such as crystal lattice spacing, mean dislocation spacing, precipitate and inclusion sizes and spacing, and grain size. The distinction based on the value of ΔK_I provides insight as to which features may be encountered in a given application.

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Figure 4: Primary fracture mechanisms in steels associated with sigmoidal variation of Fatigue crack propagation rate (da/dN) with alternating stress intensity facor (ΔK) [Ritchie, 1977].

Regime A: $(da/dN) \lesssim 10^{-8}$ m/cycle

In this regime the fatigue crack growth mechanisms are non-continuum in nature, and usually a *fatigue crack propagation threshold*, $\Delta K_{I\mathrm{th}}$, exists:

$\Delta K_{I\mathrm{th}}$ — threshold value of cyclic stress intensity factor

An operational definition of $\Delta K_{I\mathrm{th}}$ is the ΔK_I corresponding to a growth rate $\frac{da}{dN}$ of 10^{-10} m/cycle.

If $\Delta K_I < \Delta K_{I\mathrm{th}}$ then

$$\left(\frac{da}{dN}\right) \lesssim 10^{-10} \text{ m/cycle or } \left(\frac{da}{dN}\right) \approx 0 \quad \text{— i.e., a non-propagating crack.}$$

In Regime A (also known as the Threshold Regime) the crack growth rate is *sensitive to the microstructure, the R ratio,and the environment.* As shown shematically in the figure, $\Delta K_{I\mathrm{th}}$ varies widely, but for many metallic materials lies in the range $\sim 2MPa\sqrt{m} \leq \Delta K_{I\mathrm{th}} \leq \sim 10MPa\sqrt{m}$.

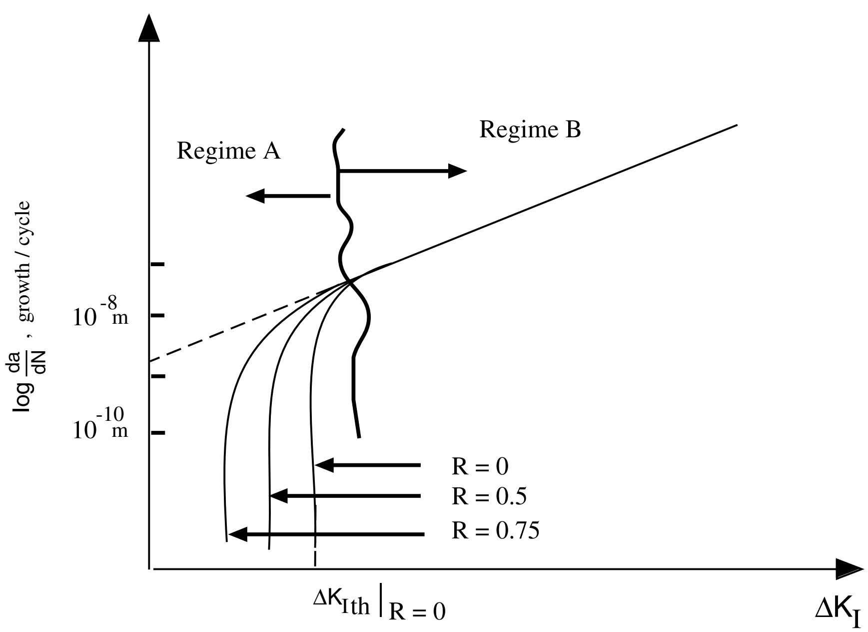

Figure 5: Schematic near-threshold fatigue cracking.

Regime B: $10^{-8} \lesssim (da/dN) \lesssim 10^{-5}$ m/cycle

In this regime, for a given value of R ratio, there is an essentially linear relationship between $\log\left(\frac{da}{dN}\right)$ and $\log(\Delta K_I)$:

$$\begin{aligned}\log_{10}\left(\frac{da}{dN}\right) &= \log_{10} A + m\log_{10}(\Delta K_I)\\ &= \log_{10} A + \log_{10}\{(\Delta K_I)^m\}\\ &= \log_{10}\{A(\Delta K_I)^m\}\end{aligned}$$

or

$$\frac{da}{dN} = A(\Delta K_I)^m.$$

In this equation, "A" and "m" are experimentally determined material constants describing the straight line portion of the $\frac{da}{dN}$ vs. ΔK_I curve. Over a broad spectrum of engineering alloys, the range of the dimensionless exponent m is $\sim 2 \leq m \leq\sim 12$, with a "typical" value of $m \simeq 4$. In Regime B (also known as the "Power Law" or "Continuum" Regime) *there is relatively little influence of microstructure, R-ratio, or dilute environment* on the fatigue crack growth behavior, and hence, on the constants A and m. The power law form of fatigue crack growth law was first proposed by Paris, Gomez, and Anderson, and is often referred to as the "Paris law."

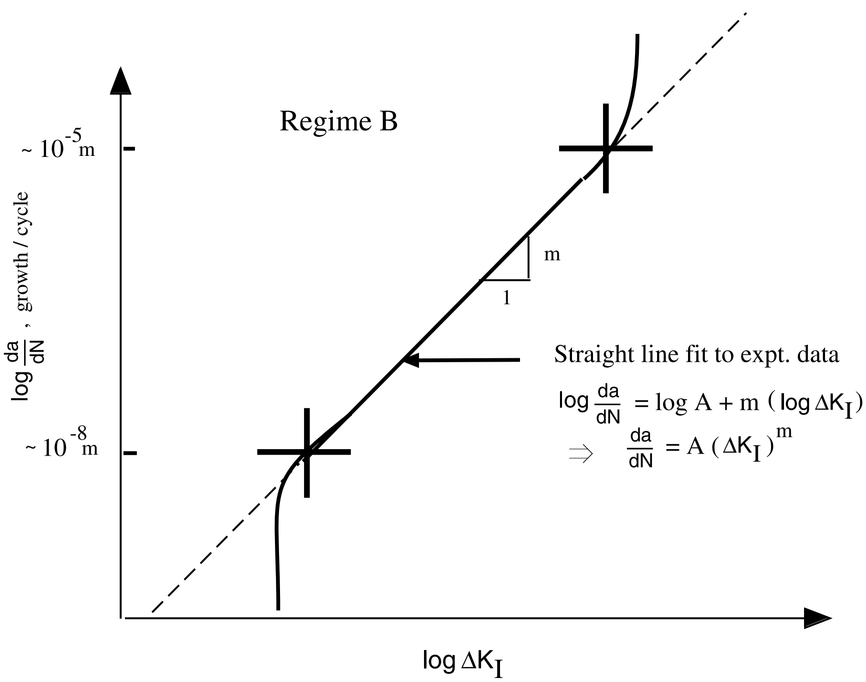

Figure 6: Fatigue crack growth rate in the power-law regime.

Regime C: $(da/dN) \gtrsim 10^{-5}$ m/cycle

In Regime C the crack growth rates are very high, and consequently little fatigue crack growth life is involved. Region C has the least importance in most fatigue situations. In this region the stress levels are high enough ($K_{\max}$ approaches K_{Ic} in Regime C) so that crack extension due to the static modes of failure like cleavage and microvoid coalescence is superposed onto the mechanism of cyclic subcritical crack extension. Because the static fracture modes are sensitive to microstructure and stress state, the growth rates in Regime C (also known as the "Static Modes" Regime) are *sensitive to the microstructure, the R-ratio and specimen thickness*. However, because (da/dN) in this regime is so high, it is *insensitive to the environment and the frequency.*

For crack–tolerant design procedures the log $\left(\frac{da}{dN}\right)$ versus $\log(\Delta K_I)$ curve is approximated as

If $\Delta K_I < \Delta K_{I\text{th}}$ then $(da/dN) = 0$
If $\Delta K_I \geq \Delta K_{I\text{th}}$ then $(da/dN) = A(\Delta K_I)^m$

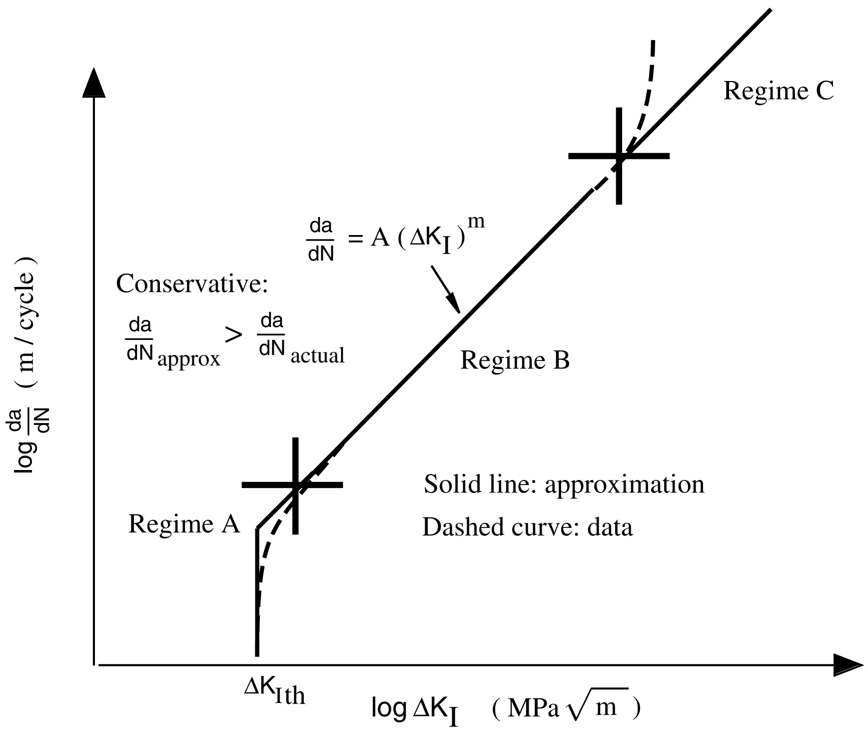

Figure 7: Rapid fatigue cracking in the "static modes" regime.

where A and m are experimentally determined constants.

$$\frac{da}{dN} = \left\{ \begin{array}{l} 0 \text{ if } \Delta K_I < \Delta K_{I\text{th}} \\ A(\Delta K_I)^m \text{ if } \Delta K_I \geq \Delta K_{I\text{th}} \end{array} \right\}$$

In the equation $(da/dN) = A(\Delta K_I)^m$,

- (da/dN) has units of (m/cycle), and
- $(\Delta K_I)^m$ has units of $(\text{MPa}\sqrt{\text{m}})^m$.

Hence,

$$A \quad \text{has strange units of} \quad \frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m} \; !$$

To make life simpler, let

ΔK_{Io}	be a reference crack driving force, and let
$\left(\frac{da}{dN}\right)_o \equiv \Delta a_o$	be the corresponding reference crack growth rate.

That is, ΔK_{Io} and Δa_o are the values of *any* point on the power law growth rate curve. Then the power law expression

$$\left(\frac{da}{dN}\right) = A(\Delta K_I)^m$$

may be written simply as

$$\left(\frac{da}{dN}\right) = \Delta a_o \left(\frac{\Delta K_I}{\Delta K_{Io}}\right)^m , \tag{1}$$

where the material constants

$$\left(\frac{da}{dN}\right)_o = \Delta a_o, \quad \Delta K_{Io}, \quad m$$

have more familiar dimensions. In applying eq. (1), it is understood that the driving force for cyclic crack growth is the cyclic stress intensity factor

$$\Delta K_I = Q \Delta \sigma \sqrt{\pi a}; \quad Q = \hat{Q}(a).$$

This last expression is dimensionally misleading since Q is dimensionless, while $[a]$ = length. Rather, it is intended to remind us of the possible functional dependence of Q on variable crack length in a structure of fixed geometry (e.g., width w). In general, $Q = \hat{Q}(a/w)$, etc.

Integration of Crack-Growth Equation

By rearranging (1), we have the differential expression

$$dN = \frac{(\Delta K_{Io})^m}{\Delta a_0} \frac{da}{(\Delta K_I)^m} ,$$

which can be integrated (on the left with respect to N and on the right with respect to a) as

$$N_{a_i \to a_f} \equiv \int_0^{N_{a_i \to a_f}} dN = \frac{(\Delta K_{Io})^m}{\left(\frac{da}{dN}\right)_o} \int_{a_i}^{a_f} \frac{da}{\left\{\hat{Q}(a) \Delta \sigma \sqrt{\pi a}\right\}^m}.$$

In writing the integrated form, we have emphasized that $N_{a_i \to a_f}$ is the number of cycles required to grow a fatigue crack from initial value "$a = a_i$" to final crack length "$a = a_f$" under the application of cyclic stress range $\Delta\sigma$ in a material having power-law fatigue crack growth behavior. We have accounted for the dependence of ΔK_I on a by substituting the stress intensity factor calibration $\Delta K_I = Q\Delta\sigma\sqrt{\pi a}$ under the integral sign.

For constant $\Delta\sigma$, this term can also be factored outside the integral:

$$N_{a_i \to a_f} = \frac{1}{\Delta a_o} \frac{(\Delta K_{Io})^m}{(\Delta\sigma\sqrt{\pi})^m} \int_{a_i}^{a_f} \frac{da}{\left\{\hat{Q}(a)a^{1/2}\right\}^m}. \tag{2}$$

In general, $\hat{Q}(a)$ is a complex function of the crack length, and it is usually necessary to perform the integration numerically. However, if Q is constant, independent of a, then (2) reduces to

$$N_{a_i \to a_f} = \frac{1}{\Delta a_o} \frac{(\Delta K_{Io})^m}{(Q\Delta\sigma\sqrt{\pi})^m} \int_{a_i}^{a_f} a^{-\frac{m}{2}} da. \tag{3}$$

Assuming a_i is known, we may define the initial range of cyclic stress intensity factor as $\Delta K_{Ii} \equiv Q\Delta\sigma\sqrt{\pi a_i}$. Thus, on multiplying the numerator and denominator by $a_i^{m/2}$, we obtain

$$N_{a_i \to a_f} = \frac{a_i^{m/2}}{\left(\frac{da}{dN}\right)_o} \left(\frac{\Delta K_{Io}}{\Delta K_{Ii}}\right)^m \int_{a_i}^{a_f} a^{-\frac{m}{2}} da. \tag{3a}$$

Finally, on integrating (3a), we obtain (for $m > 2$)

$$N_{a_i \to a_f} = \left(\frac{a_i^{m/2}}{\Delta a_o}\right) \left(\frac{\Delta K_{Io}}{\Delta K_{Ii}}\right)^m \left[\frac{1}{\left(-\frac{m}{2}+1\right)} a^{-\frac{m}{2}+1}\right]_{a_i}^{a_f}$$

$$N_{a_i \to a_f} = \left(\frac{a_i^{m/2}}{\Delta a_o}\right) \left(\frac{\Delta K_{Io}}{\Delta K_{Ii}}\right)^m \left(-\frac{2}{m-2}\right) \left[a_f^{-\frac{(m-2)}{2}} - a_i^{-\frac{(m-2)}{2}}\right],$$

or

$$N_{a_i \to a_f} = \frac{a_i^{m/2}}{\Delta a_o} \left(\frac{\Delta K_{Io}}{\Delta K_{Ii}}\right)^m \frac{2}{(m-2)} \left[\frac{1}{a_i^{\frac{(m-2)}{2}}} - \frac{1}{a_f^{\frac{(m-2)}{2}}}\right].$$

This last expression may be re-arranged as

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_o} \left(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^m \frac{2}{(m-2)} \left[1 - \left(\frac{a_i}{a_f}\right)^{\frac{(m-2)}{2}}\right]; (m > 2). \tag{4a}$$

It is sometimes useful to plot the equation for $a(N)$, based on constant $Q\,\Delta\sigma$ and an initial crack length a_i; for $m > 2$, (4a) can be inverted to give

$$a(N) = \frac{a_i}{\left(1 - \frac{N}{N_0}\right)^{\frac{2}{(m-2)}}},$$

where

$$N_0 \equiv \frac{a_i}{\Delta a_o} \left(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^m \frac{2}{(m-2)}.$$

The integral of (3) for the special case of the power-law exponent $m = 2$ provides the logarithmic form

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_o} \left(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^2 \left[\ln\left(\frac{a_f}{a_i}\right)\right] \quad (\text{for} \quad m = 2). \qquad (4b)$$

The compact expressions (4a,4b) give the number of cycles $N_{a_i \to a_f}$ required to propagate a crack from any initial size a_i to any final size a_f under conditions of constant Q and constant $\Delta\sigma$. The structure of the dependencies of $N_{a_i \to a_f}$ on the system variables is extremely revealing. In particular, the dominant role of a_i is clearly seen for $m > 2$. Also, the essential futility of improving fatigue crack propagation life by increasing toughness is evident. For $m > 2$, we see that the fatigue life is bounded, even if the material were to be made "infinitely" tough, with critical crack size $a_c = a_f \to \infty$. Similarly, the weak logarithmic improvement in fatigue crack propagation life with increasing $a_c = a_f$ for $m = 2$ is likewise indicative of little benefit of increased toughness on fatigue crack propagtion life. Finally, the nth-power dependence of growth-rate on $\Delta\sigma$ (or on Δ[load], in general) is directly reflected in a corresponding inverse power-law dependence of fatigue crack propagation like on cyclic load range.