

Rotation to Fluid Flow

Another common phenomenon in machinery is the transport of fluids. Pumps are among the more widespread machine components, as may be seen by examining a typical automobile engine.

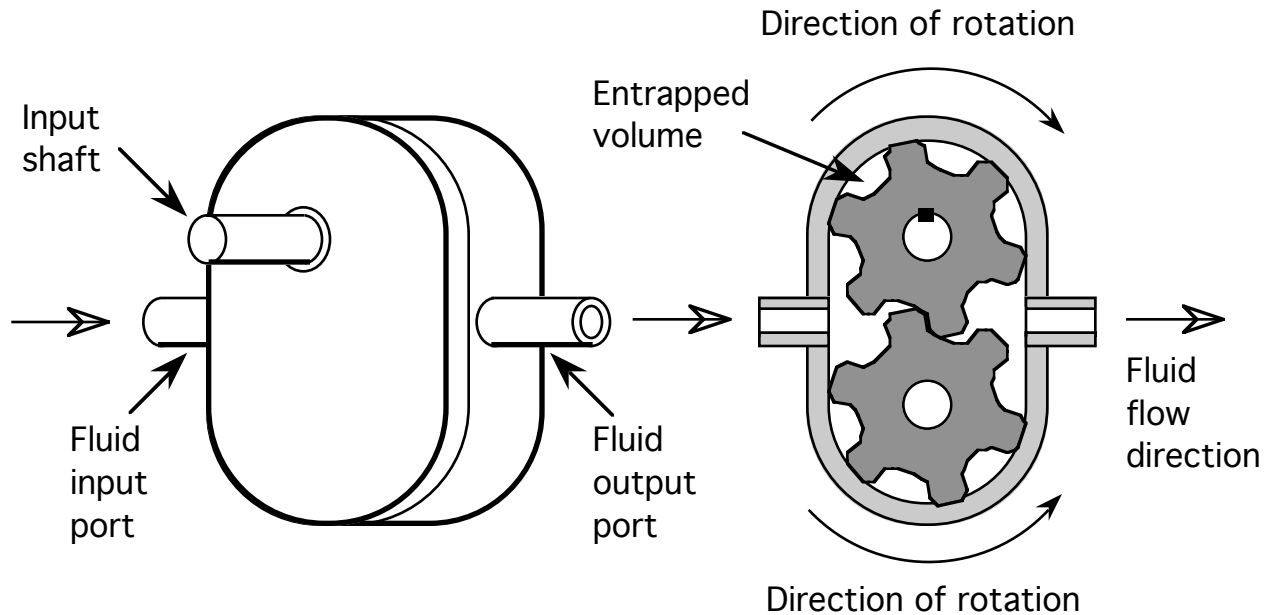


Figure 5.11: A sketch of a gear pump is shown on the left. A cross section is shown on the right.

*Gear Pump*

One of the more robust and versatile types of pump is the gear pump. As illustrated in figure 5.11, two meshing gears rotate inside a close-fitting housing. Fluid entrapped between the gears and the housing is drawn from the input port to the output port. A gear pump is generally used to pump incompressible fluids and, by design, is a *positive-displacement* pump: aside from a small amount of leakage between the gears and the housing, a fixed volume of fluid is pumped per unit rotation of the drive shaft. If we neglect any leakage in the device sketched in figure 5.11, for every sixty degrees of shaft rotation, each of the two gears pumps the volume,  $V_e$ , entrapped between one gear-tooth and the housing. The relation between displacements is as follows.

$$V = \frac{6 V_e}{\pi} \theta = K \theta \tag{5.25}$$

where  $V$  is fluid volume,  $\theta$  is shaft angle in radians and  $K$  is a constant. A relation between flows is obtained by differentiating this relation between displacements with respect to time.

$$Q = K \omega \tag{5.26}$$

where  $Q$  is volumetric flow rate and  $\omega$  is angular speed. This is one of the equations of a transformer relating the fluid flow and rotational domains. The other equation relates the pressure difference,  $P$ , between the input and output fluid ports to the moment,  $\mu$  applied to the drive shaft. It could be obtained by calculating the pressure-induced forces on the faces of the gear teeth and taking moments about the shaft axis, but that requires careful thought and tedious calculation. A far

simpler method is to remember that power into an ideal transformer must equal power out of it ( $PQ = \mu\omega$ ) thus the relation between efforts must be as follows.

$$\mu = K P \quad (5.27)$$

### *Hydraulic Motor*

An important feature of all power transduction devices is that, in general, *power may be transferred in both directions*. A transformer or gyrator is fundamentally a two-way device. Consequently, the positive-displacement gear pump depicted in figure 5.11, designed to transmit power from the rotational to the fluid domain, is also one of the basic designs for a hydraulic motor which transmits power from the fluid to the rotational domain<sup>1</sup>.

### *Parasitic Dynamics*

In addition to the power transmission it is designed for, a real pump or motor is likely to exhibit energy storage and dissipation phenomena. In the rotational domain, friction and the inertia of the rotating components may be significant. A single speed is associated with the inertia, friction and the rotational side of the pump and these elements are connected by a one-junction.

In the fluid domain, there are at least three distinct sources of energy dissipation, each associated with a different flow rate. The first is due to leakage, e.g. between the gears and the housing end-plates. This leakage flow is not proportional to the speed of the shaft, but is related to the pressure difference between input and output. If the leakage flow is not zero, then the total flow through the pump is different from the component due to shaft rotation, and each of these two flows may have a distinct dissipation phenomenon associated with it. For example, the orifice head losses at entry and exit are related to total flow while the head loss due to flow past the circular walls of the chamber is related to the flow component due to shaft rotation. The required junction structure is shown in figure 5.12.

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<sup>1</sup> However, a motor and a pump may differ in important aspects of their design details.

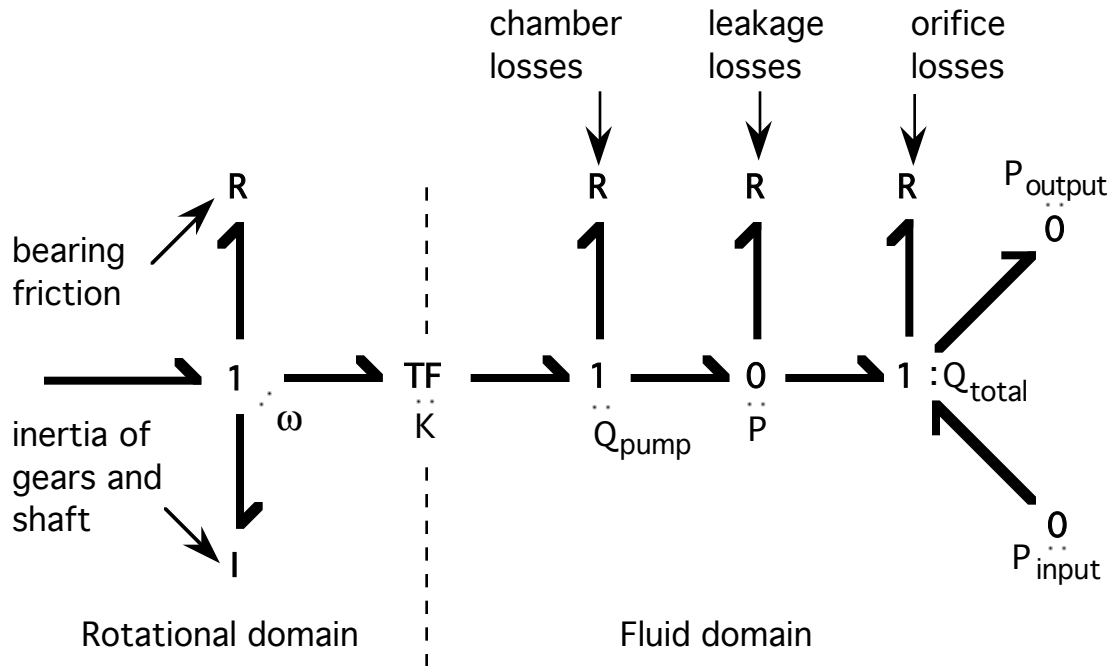


Figure 5.12: Bond graph of a model of the gear pump of figure 5.11 including plausible energetic phenomena in the rotational and fluid domains.

As before, this is not necessarily a better model than the ideal transformer, merely more detailed. The extra detail comes with a price: to be useful, the parameters of the additional elements must be determined and that may require considerable effort.