Matt Lang, MIT

$kT := 4.1 \cdot 10^{-21} \cdot J$

the viscosity is η

$\eta := 10^{-5} \frac{kg}{cm \cdot s}$

For a microscopic object moving through a viscous medium, the force required to move the object will be proportional to the length of the object, the viscosity, the speed and a factor relating to the shape: looks like a frictional force?

$\eta = 1 \times 10^{-3} \frac{kg}{m \cdot s}$

Translational drag on a sphere, holds for low Reynold's number

$a := 1 \cdot 10^{-4} \cdot cm$ typical radius

$\nu := 2 \cdot 10^{-3} \cdot \frac{cm}{s}$ typical speed

the drag coefficient f is:

$f_{sphere} := 6 \cdot \pi \cdot \eta \cdot a$

$f_{sphere} = 1.885 \times 10^{-8} \frac{kg}{s}$

The force on this sphere is given by:

$F_{sphere} := f_{sphere} \cdot \nu$

$F_{sphere} = 3.77 \times 10^{-13} N$

The Reynold's number is the ratio of intertial forces to viscous forces

$R_{reynolds} := \frac{\rho \cdot a \cdot \nu}{\eta}$

$\rho := 10^{-3} \frac{kg}{cm^3}$

where ρ is a density, the density of water

$R_{reynolds} = 2 \times 10^{-5}$

we are in the low Reynolds # range

For your reference, drag coefficients from Howard Berg's book for drag on objects of other shapes

translational drag of a disk:

where a is the radius of the disk

disk moving at random

$f_{disk\_rand} := 12 \cdot \eta \cdot a$

$f_{disk\_rand} = 1.2 \times 10^{-8} \frac{kg}{s}$

disk moving edge on

$f_{disk\_edgeon} := \frac{32}{3} \cdot \eta \cdot a$

$f_{disk\_edgeon} = 1.067 \times 10^{-8} \frac{kg}{s}$

disk moving face on

$f_{disk\_faceon} := 16 \cdot \eta \cdot a$

$f_{disk\_faceon} = 1.6 \times 10^{-8} \frac{kg}{s}$

Ellipsoid motion, lengthwise, sidewise, random

a is long axis, b is narrow one

note a >> b

say $b := \frac{a}{10}$

$$f_{ellipsoid\_lengthwise} := \frac{4\cdot\pi\cdot\eta\cdot a}{\ln\left(2\cdot\frac{a}{b}\right) - \frac{1}{2}}$$

$$f_{ellipsoid\_lengthwise} = 5.035 \times 10^{-9}\ \frac{kg}{s}$$

$$f_{ellipsoid\_sidewise} := \frac{8\cdot\pi\cdot\eta\cdot a}{\ln\left(2\cdot\frac{a}{b}\right) + \frac{1}{2}}$$

$$f_{ellipsoid\_sidewise} = 7.19 \times 10^{-9}\ \frac{kg}{s}$$

Howard also has tables for rotational drag

Einstein-Smoluchowski relation

$$D_r := \frac{kT}{f_{sphere}}$$

connects the macroscopic world of diffusion to the microscopic world of frictional drag

$$D_r = 2.175 \times 10^{-13}\ \frac{m^2}{s}$$

this = 10^{-9} $cm^2/2$

So given a drag relationship, we can use the Einstein relation to determine a diffusion constant

$$time\_diffuse(x) := \frac{x^2}{2\cdot D_r}$$

time to go 100um or so, across a screen in a microscope

$$time\_diffuse\left(10^{-4}\cdot m\right) = 2.299 \times 10^{4}\ s$$

Matt Lang, MIT

Worm Like Chain WLC interpolation

Contour length $L_c := 1500$

$Temp := 300$

Persistence Length $L_p := 53$

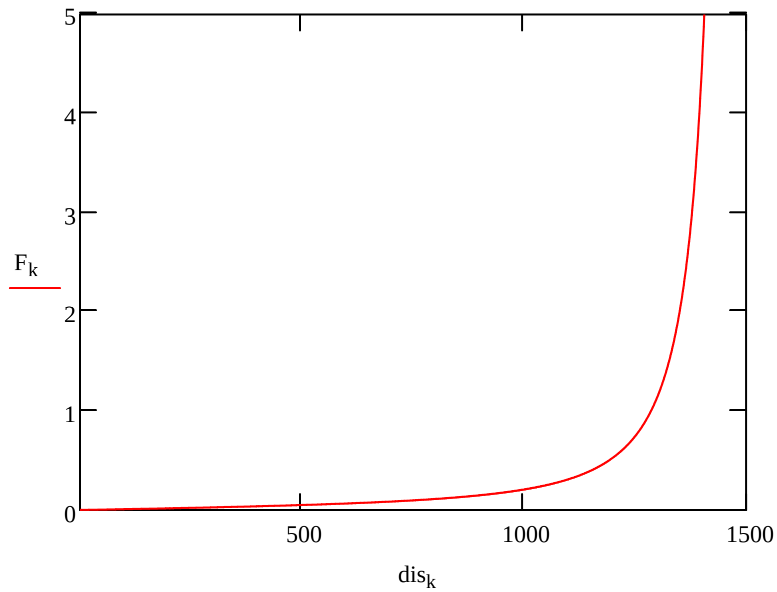
$m := 3000$

$kt := 4.1 \cdot \frac{Temp}{300}$

$k := 2 .. L_c$

$dis_k := k - 1$

$$F_k := \frac{kt}{L_p} \cdot \left[\frac{1}{4} \cdot \left(1 - \frac{dis_k}{L_c} \right)^{-2} - \frac{1}{4} + \frac{dis_k}{L_c} \right]$$



For more references, See:

Stretching DNA with Optical Tweezers, Wang et al, Biophysical Journal 72, 1335-1346 (1997)

Single M13 Bacteriophage tethering and stretching, Khalil et al, PNAS p4892-4897 (2007)