MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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2.29 NUMERICAL FLUID MECHANICS— SPRING 2007

Problem Set 1

Posted 02/08/07, due Thursday 4 p.m. 02/23/07

Problem 1.1 (40 Points)

Solve the below problems from "Chapara and Canale" textbook:

- 1.11
- 2.13, 2.15: Print the program and the result
- 3.1, 3.7, 3.9
- 4.5, 4.8, 4.15, 4.19

Problem 1.2 (10 Points)

Review MATLAB help on these commands:

- realmin
- realmax
- eps

Now use the eps for both single and double accuracy levels and compute the below series by calling a RECURSIVE function. Compute the relative and absolute accuracy and compare it by errors associated with numerical representation in your computer.

$$2 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + ...$$

Problem 1.3 (10 Points)

For a uniform inviscid flow passing a sphere with radius "R", the potential field is given by:

$$\phi(r,\theta) = U(r + \frac{R^3}{2r^2})\cos(\theta)$$

Here U is the far field velocity and the far field pressure is zero.

a) Compute the fluid velocity and plot it.

b) Discuss and evaluate the boundary conditions in infinity and in r=R.

c) Compute the pressure field and plot it.

d) Compute the drag force.

Problem 1.4 (30 Points)

The sec(x) is defined as the inverse of cos(x):

$$\sec(x) = \frac{1}{\cos(x)}$$

a) Drive the Taylor series for both sec(x) and cos(x) around x=0. Note that the coefficients of sec(x)'s Taylor series are somehow complicated so you might need a program to evaluate them.

b) Write down the numerical value of 1st eight coefficients of sec(x)'s Taylor series.

c) Now compute the by its Taylor's series expansion. Sum up the series from the 1st term until the nth term; when the last term is smaller than "esp". Report "n" beside absolute and relative error.

d) Discuss whether the error is smaller than "eps" or not. Now repeat the part c; but stop the summation when for the first time, the last nonzero term is smaller than "esp" and is approximated by zero in your computer.

e) Repeat part c, but this time stop the summation when for the 1st time adding a nonzero term does not change the value of sec(x). Can you somehow approximate an upperbound for the error just from the numerical representation?

f) Repeat part c, d and e when sec(x) and series terms are represented by "single" data type (instead of default MATLAB data type, "double", which were used in previous parts).

g) Repeat part c, d and e; but this time, with known n, sum up the series from n to 1. Do you get better results? Discuss.

In practice the summation stops whenever either the condition e or the condition c (more conservatively d) is satisfied. From now on implement both conditions at the same time for computing both sec(x) and cos(x).

h) Compute sec(x) by inversing the cos(x). However, compute the cos(x) from its series expansion. Report "n" as well as absolute and relative errors.

i) Which way of computing sec(x) is better? Discuss. If we want to use a reverse summation scheme; can you estimate the n by which the summation should start?

j) Compare n and errors for computing sec(x) by its own series versus cos(x) series for cases when $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{\pi}{16}, \frac{7\pi}{16}$.