MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
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2.29 NUMERICAL FLUID MECHANICS— SPRING 2007

Quiz 2

Takehome 48 hours, Totally 25 points
Due Thursday 4 p.m. 05/17/07, Focused on Lecture 12 to 25 (Last Lecture)

Note that you are not allowed to collaborate or share your thoughts about the problems. Please state your assumptions and write down clearly what you think about the problems even if you cannot solve them to the endpoint. Furthermore, note that we do not need you to attach your codes and we will not look through them to find what you have done, instead explain your method.

Problem 1 (5 points):

The boundary layer equation for the self-similar incompressible flow over a flat plate can be cast as the following equation and boundary conditions set:

$$f''' + \frac{1}{2} f'' f = 0, \ \textit{where}\ f''' = \frac{d^3 f}{dx^3}, f'' = \frac{d^2 f}{dx^2}$$

$$\begin{cases} f(0) = 0 \\ f'(0) = 0 \\ f'(\infty) = 1 \end{cases}$$

1. Solve the equation with your method of choice and plot the f(x) curve.
2. Find at which "x" the f' value becomes equal to "0.99" . A minimum accuracy equal to 0.1% is expected.

Problem 2 (5 points):

The steady state, fully developed laminar viscid flow in a rectangular channel with square cross section can be represented by this equation:

$$\nabla^2 u(x,y) = u_{xx} + u_{yy} = -1,\ |x|<1, |y|<1$$
$$\begin{cases} u(\pm 1, y) = 0 \\ u(x, \pm 1) = 0 \end{cases}$$

1. Assume an approximate solution of the form $U(x,y) = a_0 \cos(\frac{\pi}{2}x)\cos(\frac{\pi}{2}y)$ and find a_0 value by Galerkin's method.
2. Evaluate the normalized flow rate value ($Q^* = \int_A U(x,y)dA = \int_{-1}^{1}\int_{-1}^{1} U(x,y)dxdy$) and compare it to the analytical value which is equal to $Q^*_{analytical} = 0.5623$.
3. (EXTRA CREDIT 2 Points) Evaluate the normalized maximum shear stress ($\tau^* = \max(|\nabla U(x,y)|)$) and compare it to the analytical value which is equal to $\tau^*_{analytical} = 0.675$.

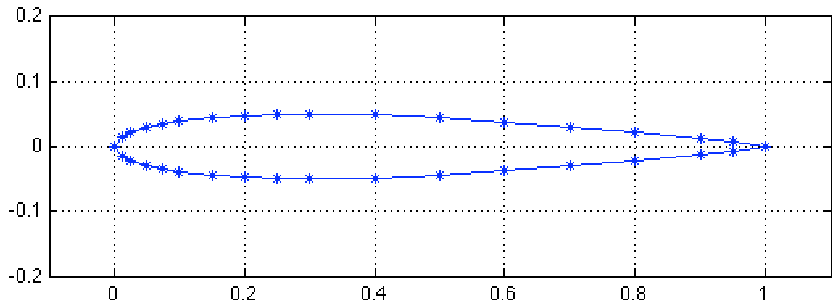
Problem 3 (5 points):

The velocity of air passing through the surface of an airfoil has been measured at different distances from the surface and has been reported in the below table. Assume that air viscosity is given by $\mu = 1.65 \times 10^{-5} \frac{\text{N} \times \text{sec}}{\text{m}^2}$ and compute the shear stress over the surface.

y(mm)	Velocity(m/s)
0	0
2	31.6
5	107.9
8	233.5
15	350.6

Problem 4 (10 points):

You are provided with "NACA0010.mat" file, which includes a discretized model of a standard symmetric NACA airfoil shown here:



1. Use simple 2D sources and solve the potential flow passing the above airfoil (assume that the angle of attack is zero). You are welcomed to use scripts given on the class but please write a few lines describing your work and the background math.
2. Plot velocity contours around the airfoil
3. Plot pressure contours around the airfoil.
4. Compute the total drag force on the airfoil
5. (EXTRA CREDIT 2 Points) Is your method applicable when the angle of attack is not zero? Explain why and provide alternatives if needed.