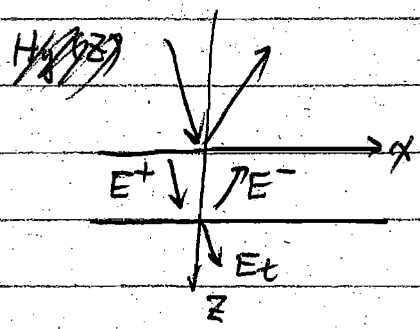
~~Exercise~~ Review of Lecture 9



(1) One layer of film

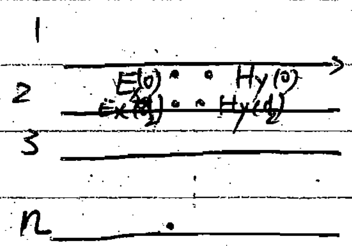
resultant wave method.

$$\Rightarrow\ r = \frac{r_{12} + r_{23} e^{2i\varphi_2}}{1 + r_{12} r_{23} e^{2i\varphi_2}}.$$

$$\varphi_2 = \frac{\cos\theta_2\, d N_2 \omega}{c_0} = \frac{2\pi \cos\theta_2 N_2}{\lambda_0}$$

Interference & tunneling phenomena.

(2) Multilayer



$$\begin{pmatrix} E_x(z) \\ H_y(z) \end{pmatrix} = \begin{pmatrix} \cos\varphi(z) & i p_2 \sin\varphi(z) \\ \frac{i}{p_2}\sin\varphi(z) & \cos\varphi(z) \end{pmatrix} \begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix}$$

$$\begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix} = \begin{pmatrix} \cos\varphi_2 & -i p_2 \sin\varphi_2 \\ -\frac{i}{p_2}\sin\varphi_2 & \cos\varphi_2 \end{pmatrix} \begin{pmatrix} E_x(d_2) \\ H_y(d_2) \end{pmatrix}$$

$$= M_2 \begin{pmatrix} E_x(d_2) \\ H_y(d_2) \end{pmatrix}$$

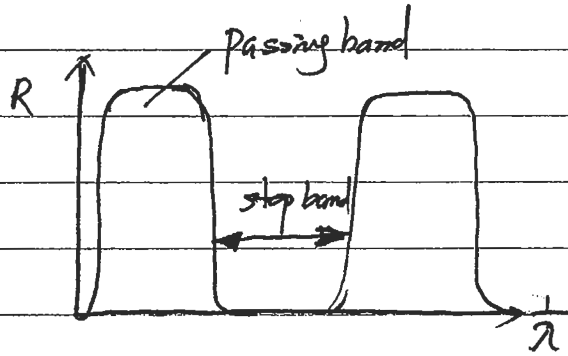
$$= M_2 \begin{pmatrix} E_x(z_3=0) \\ H_y(z_3=0) \end{pmatrix} = M_2 M_3 \begin{pmatrix} E_x(d_3) \\ H_y(d_3) \end{pmatrix}$$

$$= M_2 \cdots M_n \begin{pmatrix} E_x(d_n) \\ H_y(d_n) \end{pmatrix}$$

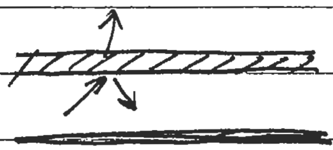
$$A \begin{pmatrix} E_i \\ E_r \end{pmatrix} = M_2 \cdots M_n\, B\, E_t$$

$$A = \begin{pmatrix} 1 & 1 \\ \frac{1}{p_1} & -\frac{1}{p_1} \end{pmatrix} \Rightarrow \text{Solve for } E_t/E_i,\ E_r/E_i$$

10/12



* transiti to thick films
* Combination of Thin & Thick films

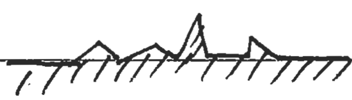


← Thin film: Interference

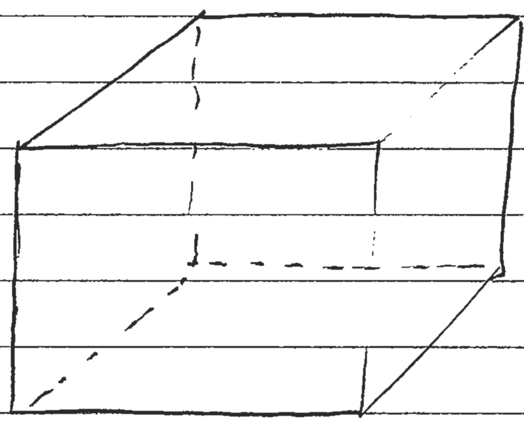
← thick film: no interference

use ray tracing result.

* roughness Interfaces.



* Planck's law : a not so rigorous proof.



Consider a cube of length L
perfectly reflecting surface
large $\Rightarrow$ free space

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

$$k_x = \pm\frac{2\pi}{\lambda_x} = \pm\frac{2\pi n_x}{L}$$

$$n_x \lambda_x = L$$

Similarly k_y, k_z.

each combinat. of n_x, n_y, n_z. is a different wave
at ω, there are many combinations
Quantum: each wave must ~~be~~ have an energy of

$$E_n = n\hbar\omega \qquad (n=0,1,\dots)$$

Statistical physics: $A\exp\left(-\frac{E_n}{kT}\right)$ = probability of actually have E_n.
Boltzmann constant

$$\sum_{n=} A\exp\left(-\frac{n\hbar\omega}{kT}\right) = 1$$

$$A = \frac{1}{1-\exp\left(-\frac{\hbar\omega}{kT}\right)}$$

~~Probability of that each wave/wave~~ average # of photons a wave has

$$\sum n \cdot p(n) = \frac{1}{e^{\frac{\hbar\omega}{kT}}-1} = f(\omega)$$

The average energy of each wave

$$\hbar\omega f(\omega)$$

We need to find out how many waves we have at ω for a width of $d\omega$.

Volume per mode n_y, n_y, n_x change by ± 1. $\left(\frac{2\pi}{L}\right)^3$

# of mode $\frac{\cancel{4\pi}\, dk_x\, dk_y\, dk_z}{(2\pi/L)^3} \cdot 2$

density of states $D(\omega) = 2 \cdot \frac{1}{d\omega} \times \frac{L^3}{8\pi^3} \cdot dk_x dk_y dk_z$

$$= 2 \frac{V}{8\pi^3} \cdot \frac{4\pi k^2 dk}{d\omega}$$

$$k = \frac{\omega}{c} \qquad \omega = ck$$

$$D(\omega) = \frac{V \omega^2}{c^3 \pi^2}$$

so, energy density per unit volume

$$U(\omega) = \hbar\omega\, f(\omega)\, D(\omega) = \frac{\hbar \omega^3}{\pi^2 c^3 (e^{\frac{\hbar\omega}{kT}} - 1)}$$

Intensity $\frac{c}{4\pi} U(\omega) = I(\omega) = \frac{\hbar \omega^3}{4\pi^3 c^2 (e^{\frac{\hbar\omega}{kT}} - 1)}$