Review of last lecture (2 lectures):

1. $E_f - E_i = \hbar\omega_p$

$\hbar\vec{k}_p = \vec{k}_f - \vec{k}_i + \vec{G}$

atoms — $\left\{ \begin{array}{l} \text{translat.} \\ \text{electronic} \end{array} \right.$

molecules — vibrat.
rotational

solid — $\left\{ \begin{array}{l} \text{electronic} \\ \text{phonon.} \end{array} \right.$

2. model for n, K.

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0(1+\chi)\vec{E}$$

Polarizat. ↑ dipole moment per unit volume

Conduct. electron $\quad \vec{J} = \sigma\vec{E}$

$$\nabla\times\vec{B} = \frac{\partial\vec{D}}{\partial t} + \vec{J} \quad \Rightarrow \quad \nabla\times\vec{B} = \frac{\partial}{\partial t}\left[\varepsilon_0\underbrace{(1+\chi - \frac{\sigma}{i\omega})}_{\varepsilon_r}\vec{E}\right]$$

dipole $\qquad$ free electron

$\longrightarrow \vec{E}e^{-i\omega t}$

$\oplus$—$\ominus$ $\longrightarrow x$

$$m\frac{d^2x}{dt^2} = -\beta\frac{dx}{dt} + E_0 e^{-i\omega t} - K(x - x_0)$$

$$\frac{d^2\Delta x}{dt^2} + \gamma\frac{d\Delta x}{dt} + \omega_0^2\Delta x = \frac{(-e)E_0}{m}e^{-i\omega t}$$

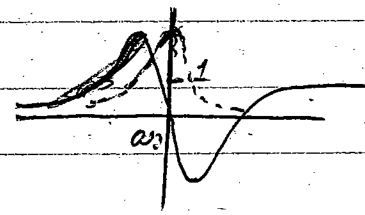
$\Delta x = x_0 e^{-i\omega t}$; $\quad x_0 = \dfrac{E_0/m}{+\omega^2 + i\gamma\omega \pm \omega_0^2}$

$$\vec{P} = Nex_0e^{-i\omega t} = \chi\vec{E}$$

$$\varepsilon_r = 1+\chi = 1+\frac{Ne^2/m}{(\omega^2-\omega_0^2)+i\gamma\omega}$$ — Lorentz model.

$$= 1+\frac{\omega_p^2}{(\omega^2-\omega_0^2)+i\gamma\omega}$$

$$= \varepsilon_r + i\varepsilon_i$$



Free electron:

$$\Rightarrow K = 0$$

$$x_0 = \frac{eE_0/m}{\omega^2+i\gamma\omega}$$

$$\vec{J} = -e\vec{v}N = \underbrace{+i\omega\frac{e^2N/m}{\omega^2+i\gamma\omega}}_{\sigma}\ \underbrace{E_0 e^{-i\omega t}}_{E}$$

free electron only

$$\varepsilon_r = 1-\frac{\sigma}{i\omega} = 1-\frac{\omega_p^2}{\omega^2+i\gamma\omega}$$ Drude model.

$$\omega_p^2 = \frac{e^2N}{m}$$

$$\sqrt{\varepsilon_r} = n+i\kappa$$

Combination of oscillators & free electron

$$\varepsilon_r = 1+\sum_n \frac{\omega_{jn}^2}{\omega^2-\omega_{0j}^2+i\gamma_j\omega} - \frac{\omega_p^2}{\omega^2+i\gamma\omega}$$

$\varepsilon_0 +$

↑ low frequency

— Drude free electron model.

$$\sqrt{\varepsilon_r} = n + i\kappa$$

$$n^2 - \kappa^2 = 1 - \frac{\omega_p^2 \omega^2}{\omega^4 + (\omega/\tau)^2}$$

$$2n\kappa = \frac{\omega^2 \omega/\tau}{\omega^4 + (\omega/\tau)^2}$$

Discussion: ① Set $\gamma = 0$

when $\varepsilon_r = 0$ $\quad 1 - \frac{\omega_p^2}{\omega^2} = 0$

$\omega_p = \omega$

at this frequency, inside the material $\vec{D} = 0$
external field is cancelled.

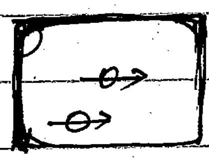
plasmon — electrons inside the material
exactly cannel external field.
longitudinal wave.

② $\omega < \omega_p$. ε_r negative
κ large.
$\Rightarrow$ highly reflecting $\quad R = \left| \frac{n + i\kappa - 1}{n + i\kappa + 1} \right|$

metal shinny

③ $\omega > \omega_p$, $\quad$ ~~ε rea~~ n, κ real
metal becomes transparent.

Local field correction



$$\vec{E}_{local} = \vec{E}_{ex} + \vec{E}_{md}$$

$$\vec{E}_{md} = \vec{P}/3\epsilon_0$$

$$\vec{E}_{ex}(\epsilon_r - \epsilon_0) = \vec{P}$$

$$\Rightarrow \vec{E}_{local} = \frac{\epsilon_r + 2\epsilon_0}{3\epsilon_0}\vec{E}_{ex}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3}\sum_j \frac{\omega_{pj}^2}{\omega_{0j}^2 - \omega^2 - i\gamma_j\omega}$$

$$= \rho \cdot \text{constant} \quad (\omega \ll \omega_0)$$

Clausius–Mossotti relation (Lorentz–Lorenz relation)