

03/02

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

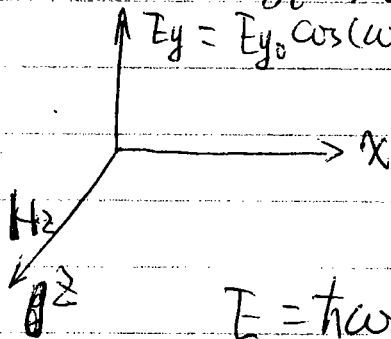
$$\nabla \cdot \vec{D} = \rho_{\text{net}}, \quad \nabla \cdot \vec{B} = 0$$

$$\vec{D} = \epsilon \vec{E}, \quad \vec{J} = \sigma \vec{E}, \quad \vec{B} = \mu \vec{H}$$

$\epsilon = \text{const}$  spatially

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2} \cdot \mu \epsilon$$

$$E_y = E_{y0} \cos(\omega t - kx)$$



$$k^2 = \epsilon \mu \omega^2$$

$$k = \frac{\omega}{\sqrt{\frac{1}{\mu \epsilon}}} = \frac{\omega}{c}$$

$$E = \hbar \omega, \quad P = \hbar k, \quad n = c_0/c = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}}$$

$$\vec{E} \perp \vec{H} \perp \vec{k}, \quad E_y = E_{y0} e^{-i(\omega t - kx)} = \sqrt{\epsilon_r}$$

$$\vec{E} = \text{Re}(\vec{E}_c)$$

$$\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}, \quad |\vec{k}| = \frac{2\pi}{\lambda}$$

$$\vec{E}_c = \vec{E}_0 \exp[-i(\omega t - \vec{k} \cdot \vec{r})], \quad |\vec{k}|^2 = \frac{\omega^2}{c^2}$$

If  $\sigma \neq 0$

$$+\vec{k} \cdot \vec{k} = +\mu \epsilon \omega^2 + i\omega \mu \sigma$$

$$\vec{k} \cdot \vec{k} = \mu \omega^2 \left[ \epsilon_0 (1 + \chi) + i \frac{\sigma}{\omega} \right]$$

$$N = \sqrt{\tilde{\epsilon}_r} = n + ik$$

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↑

↑

Complex  
refractive  
index

refractive  
index

distinction  
coefficient

$$\begin{aligned}
 E_y &= E_{y_0} \exp\left[-i\left(\omega t - \frac{N\omega}{c_0} x\right)\right] \\
 &= E_{y_0} \exp\left[-i\omega\left(t - \frac{N+ik}{c_0} x\right)\right] \\
 &= E_{y_0} e^{-\frac{\omega k}{c_0} x} \exp\left[-i\omega\left(t - \frac{N}{c_0} x\right)\right]
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow H_z &= H_{z_0} \exp\left[-i\omega\left(t - \frac{N}{c_0} x\right)\right] \\
 H_{z_0} &= \frac{N}{\mu c_0} E_{y_0} \quad (\text{positive propagation}) \\
 &= -\frac{N}{\mu c_0} E_{y_0} \quad (\text{Negative propagation})
 \end{aligned}$$

~~Proof~~

$$-\nabla \cdot (\vec{E} \times \vec{H}) = \frac{1}{2} \epsilon \frac{\partial (\vec{E} \cdot \vec{E})}{\partial t} + \frac{1}{2} \mu \frac{\partial (\vec{H} \cdot \vec{H})}{\partial t} + \vec{E} \cdot \vec{J}$$

$$\int_V dt \left\{ -\nabla \cdot (\vec{E} \times \vec{H}) \right\} = \int_V \left\{ \frac{1}{2} \epsilon \frac{\partial (\vec{E} \cdot \vec{E})}{\partial t} + \frac{1}{2} \mu \frac{\partial (\vec{H} \cdot \vec{H})}{\partial t} + \vec{E} \cdot \vec{J} \right\} dt$$

Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H} = \text{Re}(\vec{E}_c) \times \text{Re}(\vec{H}_c)$$

$$\langle \vec{S} \rangle = \frac{1}{T} \int_0^T (\vec{E} \times \vec{H}) dt = \frac{1}{2} \text{Re}(\vec{E}_c \times \vec{H}_c^*)$$

$$\langle \vec{S} \rangle = \frac{1}{2} \text{Re}(\vec{E}_c \times \vec{H}_c^*) = \frac{1}{2} \text{Re} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & E_y & 0 \\ 0 & 0 & H_z \end{vmatrix}$$

$$\Rightarrow S_x = \frac{1}{2} \text{Re}(E_y H_z^*)$$

$$= \frac{1}{2} \text{Re} \left\{ E_{y_0} \exp\left(i \frac{\omega N x}{c_0}\right) \frac{N^*}{\mu c_0} E_{y_0}^* \exp\left(-i \frac{\omega N^* x}{c_0}\right) \right\}$$

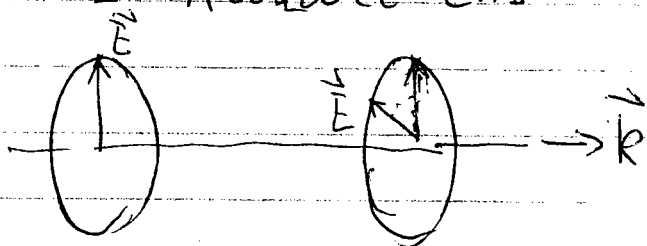
$$= \frac{1}{2} \frac{|E_{y0}|^2}{\mu c_0} \exp\left(-\frac{2\omega k}{c_0} x\right)$$

$$\uparrow -\alpha x \quad \alpha = \frac{4\pi k}{\lambda}$$

$$\delta = \frac{1}{\alpha} = \frac{\lambda_0}{4\pi k} \quad \text{--- skin depth}$$

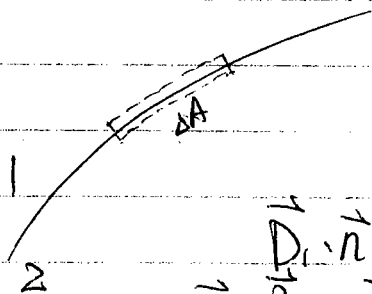
$$S = S_0 e^{-\alpha x}, \quad \dot{q} = -\frac{dS_x}{dx} (-\nabla \cdot \vec{S}) = \alpha S_0 e^{-\alpha x}$$

$$\begin{aligned} \vec{E} &= \text{Re}(\vec{E}_c) = \text{Re}\left\{(\vec{A} - \vec{B}i) e^{-i\omega t - \frac{N}{c}x}\right\} \\ &= \vec{A} \cos\left[\omega t - \frac{N}{c}x\right] - \vec{B} \sin\left[\omega t - \frac{N}{c}x\right] \end{aligned}$$



Interface conditions

$$\int (\nabla \cdot \vec{D}) dt = \int P_{net} dt$$



$$\oint \vec{D} \cdot d\vec{A} = \int P_{net} dt = \rho_s \cdot dA$$

$$\vec{D}_1 \cdot \vec{n} - \vec{D}_2 \cdot \vec{n} = \rho_s \quad (\text{ideal conductor/battery (charge)})$$

$$\vec{n}_1 \cdot \vec{B}_1 = \vec{n}_2 \cdot \vec{B}_2$$

$$(\vec{E}_1 - \vec{E}_2) \times \vec{n} = 0$$

$$(\vec{H}_1 - \vec{H}_2) \times \vec{n} = \vec{J}_s \quad (\text{charge on electrode})$$