MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.004 Dynamics and Control II
Fall 2007

Problem Set #2

Posted: Friday, Sept. 14, '07 **Due: Friday, Sept. 21, '07**

---

1. In class, we showed in two different ways that the torque constant of a DC motor equals the back–EMF constant, $K_m = K_v$. Verify from the definitions of these constants, $K_m i = T$ and $K_v \omega = v_e$, respectively, that the units associated with these constants are consistent as well.

2. Rework Problem 2 of Problem Set #1 (the motor–shaft system of Lecture 2 with non–zero initial condition) by using the Laplace transform. You should arrive at the same step response as you did in Problem Set #1.

3. Obtain the inverse Laplace transform of the following frequency–domain expressions:

a) $F_1(s) = -\dfrac{(4s-10)}{s(s+2)(s+5)}$;

b) $F_2(s) = \dfrac{4}{s^2\left(s^2+4\right)}$;

c) $F_3(s) = \dfrac{s^3-3s^2+s+2}{s}H(s)$, where $H(s)$ is a well–behaved function of s, whose inverse Laplace transform is $h(t)$.

4. Obtain the transfer function of problem 4.a in the Problem Set #1 by Laplace transforming the equations of motion that you derived previously. (If you don't have copy of your solution, you can find the equations on motions in the solutions to Problem Set #1 that have been posted on `Stellar.`) The input to the system is the force $f(t)$ and the output the rotation angle $\theta(t)$.

5. On the next page is a diagram of a DC motor connected in parallel to a current source i_s. The torque and back–EMF constants of the motor are K_m, K_v, respectively, the motor resistance is R, also modeled as connected in parallel, the motor inertia is J_m (not shown), and the motor inductance is negligible. The motor load is an inertia J with compliance K and viscous friction coefficient b, and it is attached to the motor via a gear pair with gear ratio N_1/N_2. The system input is the current i_s and the output is the rotation angle θ of the inertia. Derive the transfer function of this system.