Today's goals

- **Monday**
  - Proof that the frequency response as function of frequency ω is simply the value of the transfer function at s=jω
  - Bode plots: amplitude and phase of the frequency response on a log-log plot
  - Bode plots for elementary 1st order systems: derivative; integrator; zero; pole
- **Today**
  - Frequency response and Bode plots of underdamped 2nd order systems
  - Cascading sub-systems: rules for Bode plots of systems with multiple poles and zeros

Elementary Bode plots: 1st order

Normalized and scaled
Bode plots for
a. $G(s) = s$;
b. $G(s) = 1/s$;
c. $G(s) = (s + a)$;
d. $G(s) = 1/(s + a)$

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Please see: Fig. 10.9 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Bode plot for underdamped 2nd order system

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{1}{(\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega}$$

Note: the Bode magnitude at $\omega = \omega_n$ is

$-20\log 2\zeta$.

This can be used as correction to the asymptotic plot.

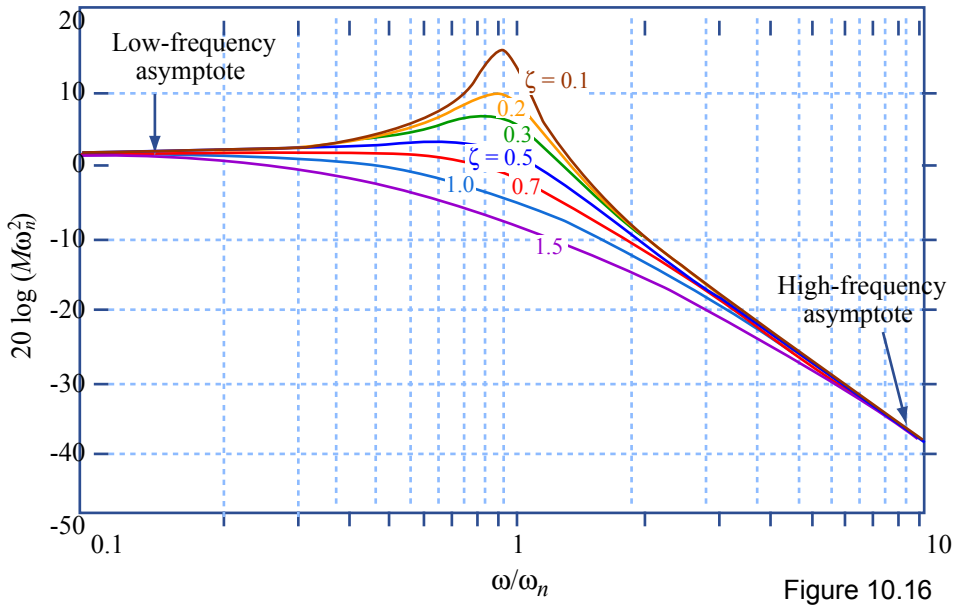

Figure 10.16

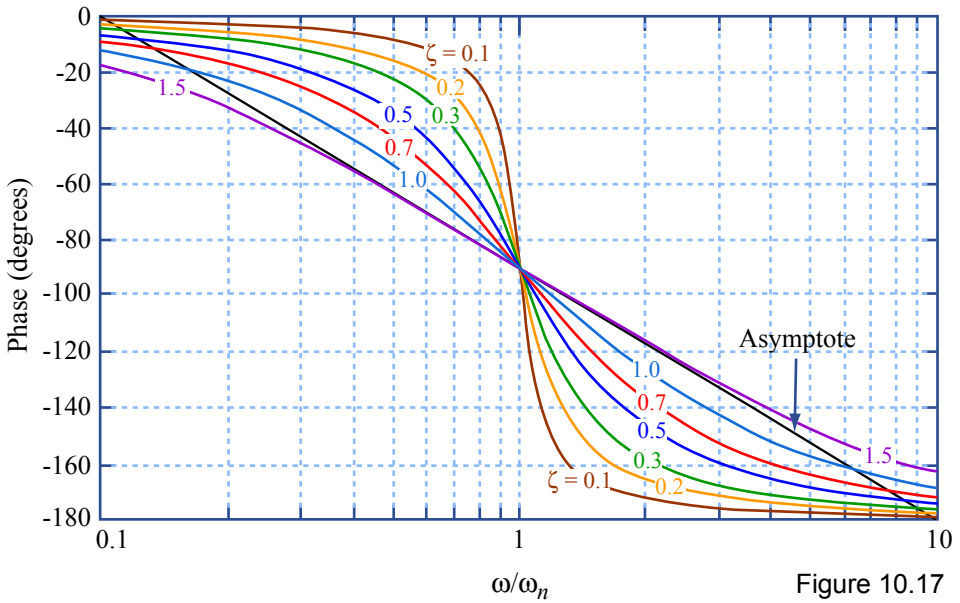

Figure 10.17

Figure by MIT OpenCourseWare.

Cascading 1st order subsystems

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)} = \frac{\frac{3}{2}K\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$

Magnitude plot

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Please see: Fig. 10.11 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Cascading 1st order subsystems

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)} = \frac{\frac{3}{2}K\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$

Phase plot

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Please see: Fig. 10.12 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Cascading 1st and 2nd order subsystems

$$G(s) = \frac{K(s+3)}{(s+2)(s^2+2s+25)} = \frac{3}{50}\frac{K\left(\frac{s}{3}+1\right)}{\left(\frac{s}{2}+1\right)\left(\frac{s^2}{25}+\frac{2s}{25}+1\right)}$$

Magnitude plot

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Please see: Fig. 10.18 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Cascading 1st and 2nd order subsystems

$$G(s) = \frac{K(s+3)}{(s+2)(s^2+2s+25)} = \frac{3}{50}\frac{K\left(\frac{s}{3}+1\right)}{\left(\frac{s}{2}+1\right)\left(\frac{s^2}{25}+\frac{2s}{25}+1\right)}$$

Phase plot

Image removed due to copyright restrictons.

Please see: Fig. 10.19 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.